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Cityscape and Land Use Regulation

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Abstract:
In 2006, the Japanese supreme court has announced that the right to enjoy the cityscape should be protected. This statement makes local governments introduce the cityscape ordinance. In most cases, the ordinance takes mainly the form of the building height regulation in the city center. As in Bertaud and Brueckner (2005), the regulation results in the spatial expansion of the urban area while it protects the amenity value. In other words, when considering the building height regulation, it is necessary to consider the tradeoff between the commuting cost and the amenity value. Based on this tradeoff, we develop a model in which the historical district exists in the city center. By using this model, we evaluate the welfare effects of the regulation. According to the comparison of the outcomes under the regulation and the laissez faire development, we obtain the following results. First, the residents strictly prefer the building regulation if the marginal disutility of the overinvestment in the district is sufficiently large. Second, the regulation always augments the aggregate land rent. Also note that, in such situation, the building height regulation may become the second-best policy.
Keywords: Cityscape, Building Height Regulation, Second-best

1. Introduction

In 2002, Tokyo district court gave a decision to a developer in Kunitachi, Tokyo. In the decision, since the developer destroyed the city scape by constructing the building higher than 20 meters, he had to remove the building above 20 meters. Historically, residents in Kunitachi set the upper limit of the building at 20 meters and Kunitachi constituted its cityscape. The court considered the developer's behavior as destroying the coordination of residents. The developer, however, made an appeal and, in 2006, the Japanese supreme court rejected the decision by Tokyo district court. Although the decision is denied, the supreme court announced that the right to enjoy the cityscape should be protected. After this announcement in 2006, several cities have introduced the cityscape ordinance. The ordinance took the form of the building height regulation at the city center. When considering the building height regulation at the city center, it is necessary to focus on the tradeoff between the high amenity value and the physical expansion of the city. Specifically, the expansion of the city augments the commuting cost of residents. Focusing on this tradeoff, this paper addresses the problem whether the building height regulation enhances the economic welfare. In addition, if it is true, we also derive the condition where the building height regulation becomes the second-best policy.

In order to deal with these two problems, we develop a traditional model of urban economics. Namely, the city is represented by a linear space and the historical district locates around the city center. Within the district, the ideal building height is predetermined. Once the average building height equals to the ideal, the value of the amenity is maximized. The amenity value improves the residents' utility and its effect declines as the difference in the ideal and the average expands. Under this setup, we deal with the two alternative situations such as: i) the government regulates the building height; and ii) developers freely construct the building within the district. Under this setup, we compare the equilibrium outcomes and obtain the following results. First, if the marginal disutility of the higher building in the district is sufficiently large, residents of the city may prefer the regulation to the laissez faire development. Furthermore, if the ideal building height is sufficiently low, the landlords also strictly prefer the building height regulation. In other words, if these two conditions are met, the building height regulation becomes the second-best.

The building height regulation itself is a topic drawing attention in the literature. Bertaud and Brueckner (2005) studies the regulation in Bangalore and it is shown that the building height regulation worsens the economic welfare compared to the laissez faire development since the benefit of the regulation is ignored. Brueckner et al. (1999) considers the urban amenity located at the city center and evaluates its effect on the residence choice by the heterogeneous income group. With respect to the factors that affect the residence choice, there are several studies. For example, Brueckner and Rosenthal (2009) shows that, due to the history of the urban development, the rich prefers suburb because the suburb has the rich new dwellings. Glaeser et al. (2008) explains the accessibility to the public transportation is one of the key factors that attracts the poor residents to the city center. In other words, the urban amenity takes the several forms such as the access to the public transportation mode and the quality of the housing stocks. In this paper, we focus on the historical sights as the urban amenity and the building height regulation is introduced in order to preserve the amenity. Hence, we construct a model that combines these two literatures, Brueckner et al. (1999) and Bertaud and Brueckner

(2005), and evaluates the welfare effect of the regulation.

This paper is organized as follows: Section 2 describes the model. Section 3 summarizes the equilibrium with and without the building height regulation. In order to evaluate the welfare effects of the regulation, we treat the equilibrium with the building height regulation as the benchmark. Section 4 evaluate the welfare effects of the building height regulation by comparing the outcomes obtained in Section 3 with the optimum. This section also provides the report on the incidence of the net gain between the residents and the landlords. Finally, Section 5 states the concluding remarks.

2. The Model

Suppose a linear city. The CBD locates at the left end, and we set the CBD as the origin of coordinates; therefore, each location within the city is represented by the distance to the CBD, x. In addition, we assume that at each location, x, the amount of the land is normalized to unity. In the city, we have a historical district with the range of $0 \leq x \leq b_0$, and, within the district, at $x = x_h < b_0$, a historical site exists. The historical district generates the amenity value, and the value depends on the difference in the average building height within the district, $\bar{h}$, and the ideal building height, $\bar{h}_0$. Furthremore, the amenity value decreases as the distance to the site, $|x - x_h|$, increases. For the sake of the simplicity, we assume that the historical site locates at the CBD (that is, $x_h = 0$). Formally, the amenity value is written as a function of the average building height, $\bar{h}$, and the distance to the CBD, x. It is specified as:

$$a\left(x,\bar{h}\right)=a_0 \exp\left(-\eta\left|\bar{h}-\bar{h}_0\right|-\tau x\right). \tag{1}$$

where $\eta > 0$ and $\tau > 0$ respectively shows the marginal effect on the amenity of the building height and the distance.

Let us denote by $h(x)$ the building height at x; then, the average, $\bar{h}$, is defined as

$$\bar{h}=\int_0^{b_0}\frac{h(x)}{b_0}dx.$$

As in Eq. (1), with respect to the average building height, the amenity value is maximized at $\bar{h} = \bar{h}_0$. Namely, within the historical district, $\bar{h}_0$, captures the ideal building height, and the harmonious building height generates the higher amenity. In other words, in order to maximize the value of the amenity, the government should regulate the building height within the historical district to μ: that is, $h(x) = \bar{h}_0$ for $0 \le x \le b_0$.

The land of the economy is under the absentee ownership. Landlords maximize the revenue from their lands by choosing the land use, residential or agricultural use. If they choose the agricultural use, they receive the agricultural land rent, $\bar{r}$, per unit land. In addition, without loss of generality, we normalize the agricultural land rent to zero. In case of the residential use, they lend their lands to the developers and earn the rent $r(x)$.

With respect to the developers' behavior and the building height regulation, we consider the two alternative cases: i) the government sets the building height within the historical district to $\bar{h}^R \geq \bar{h}_0$ (Regime R); ii) the developers are free to choose the height within the district under the competitive environment (Regime C). We treat Regime R as the benchmark case, and by comparing with R, we evaluate the welfare effects of the building height regulation. Hereafter, Subsection 2.1 describes the household's behavior while Subsection 2.2 summarizes the behavior of developers under the two alternative regimes.

2.1. Households

The population of the city is given by n, and no migration from/to the outside of the city is allowed. Households are homogeneous in both income and preference. They earn the wage y by commuting to the CBD. The commuting cost is linear in the distance to the CBD, x. They utilize the disposable income, $y - tx$, for the expenditures of the composite good, z, and the housing, pq. Furthermore, in order to simplify the analysis, we assume that, independent from the location, all households consume the same amount of the lot size, q. In order to simplify the analysis, we normalize the lot size to unity: that is, $q = 1$. The budget constraint for a household at x is:

$$y - tx = z + p \leftrightarrow p = y - tx - z, \tag{2}$$

where p and t respectively represent the housing rent and the commuting cost per unit distance.

The utility of a representative household depends on the consumptions of composite goods and the amenity at their residence, $a(x, \bar{h})$. Namely, the utility function is specified as:

$$U\left(z, a\left(x, \bar{h}\right)\right) = z + \ln a\left(x, \bar{h}\right) = z + \ln a_0 - \eta\left|\bar{h}_0 - \bar{h}\right| - \tau x. \quad (3)$$

Since households are homogeneous in both the preference and the income, independent from the residence, they attain the same utility level, u, at the location equilibrium. Hence, given the utility level, u, the consumption of the composite goods at x is computed as:

$$z^*\left(x, \bar{h}, u\right) = u - \ln a_0 + \eta\left|\bar{h}_0 - \bar{h}\right| + \tau x. \quad (4)$$

Plugging Eq. (4) into (2), the bid housing rent at x is derived as:

$$p^*\left(x, \bar{h}, u\right) = y - tx - z^*\left(x, \bar{h}, u\right) = y + \ln a_0 - \eta\left|\bar{h}_0 - \bar{h}\right| - u - x\left(t + \tau\right) \quad (5)$$

Differentiating Eq. (5) with respect to the distance to the CBD, x, and the average building height, $\bar{h}$,

$$\frac{\partial p^*\left(x, \bar{h}, u\right)}{\partial x} < 0 \text{ and } \operatorname{sgn}\left(\frac{\partial p^*\left(x, \bar{h}, u\right)}{\partial \bar{h}}\right) = \operatorname{sgn}\left(\bar{h}_0 - \bar{h}\right).$$

At the boundary of the historical district, $x = b_0$, the slope of the bid housing rent changes: that is, outside the district, the slope becomes steeper than the one inside the district since the amenity value declines as the distance to the CBD increases. In contrast, the effect of the building height depends on the difference between the average and the ideal heights. In order to evaluate the welfare effects of the building height regulation, we consider the case where the average height is higher than the ideal height. In such case, the bid housing rent decreases as the developers construct the higher buildings within the district: that is, $\partial p^*/\partial \bar{h} < 0$.

2.2. Developers under the Alternative Regimes

Developers construct the housing lots by using the land and the capital. Since, at each location, the amount of the land is normalized to unity, we express the input of each developer by the capital-land ratio, S. Namely, the technology of the developer is specified as:

$$h = f(S) = \sqrt{S}. \tag{6}$$

According to Eq. (6), the output produced by the developer is interpreted as the building height: that is, the more capital implies the higher building. The total cost of developing each location x is equal to the sum of the payments to the capital and the land. We normalize the price of the capital to unity. In such case, since the land at x is also normalized to unity, the cost of developing at x is equal to $S + r(x)$ where $r(x)$ is the land rent. Hence, the developer's profit at location x is computed as:

$$\pi(x, \bar{h}, u) = p^*(x, \bar{h}, u)\sqrt{S} - S - r(x). \tag{7}$$

Under Regime R, the building height within the historical district is regulated to $\bar{h}_0$

while, outside the district, developers can freely choose the building height at each location. Formally, the problem of each developer within the district is formulated as:

$$\max_S \pi\left(x, \bar{h}^R, u\right) \text{ subject to } \bar{h}^R = \sqrt{S}. \tag{8.1}$$

Outside the historical district, the developer's problem is written as:

$$\max_S \pi\left(x, \bar{h}^R, u\right). \tag{8.2}$$

In contrast, under Regime C, independent from the location, each developer freely chooses the building height in order to maximize the profit:

$$\max_S \pi\left(x, \bar{h}^C, u\right), \tag{9}$$

where $\bar{h}^C$ is the average building height within the district under Regime C. By solving Eqs. (8) and (9), the developer's investment under Regime i $(i = R, C)$, $S^i\left(x, \bar{h}^i, u\right)$, is computed where $\bar{h}^R = \bar{h}_0$.

Finally, under each regime, due to the competition among developers, each developer at x earns zero profit. In other words, in order to overcome the competition against the potential entrants, each developer must pay all profits to landlords. Formally, the bid land rent at x is written as the function of the distance to the CBD, x, the average building height within the district, $\bar{h}^i$ $(i = R, C)$, and the utility, u: that is, $r(x) = r^i\left(x, \bar{h}^i, u\right)$. By using the bid land rent and the solutions of Eqs. (8) and (9), the boundary of the city, b^i, and the resident's utility level, u^i, are determined according to the following conditions:

$$r^i\left(b^i,\bar{h}^i,u^i\right)=0, \tag{10.1}$$

$$\int_0^{b^i} f\left(S^i\left(x,\bar{h}^i,u^i\right)\right)dx=n. \tag{10.2}$$

3. Equilibria

In this section, we show the equilibria under the two alternative situations, with and without the building height regulation. First, in Subsection 3.1, we consider Regime R in which the government regulates the building height within the historical district. In addition, we treat Regime R as the benchmark case, and, in order to evaluate the welfare effects of the regulation, we introduce the compensation, s, to each resident when the building height regulation is abolished. Under such circumstance, Subsection 3.2 focuses on the behavior of developers and derives the laissez faire equilibrium solutions. In addition, this subsection compares the outcomes under the two alternative situations and evaluates the welfare effects of the building height regulation.

3.1. The Equilibrium with the Building Height Regulation

The developer's problem differs with the location. Developers at $x \le b_0$ solve the problem (8.1) while the problem for those at $x > b_0$ is characterized by Eq. (8.2). For each situation, the solutions are computed as:

$$S^R\left(x,\bar{h}^R,u\right)=\begin{cases}\bar{h}^{R2} & \text{for } 0\le x\le b_0,\\ \dfrac{\left\{y+\ln a_0-u-x(t+\tau)-\eta\left(\bar{h}^R-\bar{h}_0\right)\right\}^2}{4} & \text{for } b_0<x,\end{cases}$$

(11.1)

$$\lambda^R\left(x,\bar{h}^R,u\right)=y+\ln a_0-u-x(t+\tau)-2\bar{h}_0-\eta\left(\bar{h}^R-\bar{h}_0\right) \text{ for } 0\le x\le b_0,$$

(11.2)

where $\lambda^R\left(x,\bar{h}^R,u\right)$ is the Lagrange multiplier for the constraint of Eq. (8.1). Since we assume that the building height regulation is binding for all the locations within the district, the Lagrange multiplier is always positive for $x\le b_0$.

Substituting Eqs. (11) into (7) and applying the zero-profit condition, we have the bid land rent as:

$$r^R\left(x,\bar{h}^R,u\right)=\begin{cases}\bar{h}^R\left\{y+\ln a_0-u-x(t+\tau)-2\bar{h}^R-\eta\left(\bar{h}^R-\bar{h}_0\right)\right\}-\bar{h}_0^2 & \text{for } 0\le x\le b_0,\\ \dfrac{\left\{y+\ln a_0-u-x(t+\tau)-\eta\left(\bar{h}^R-\bar{h}_0\right)\right\}^2}{4} & \text{for } b_0<x.\end{cases}$$

(12)

According to Eqs. (10), (11), and (12), the equilibrium conditions are given by:

$$r^R\left(b^R,\bar{h}^R,u^R\right)=\frac{\left\{y+\ln a_0-u^R-b^R(t+\tau)-\eta\left(\bar{h}^R-\bar{h}_0\right)\right\}^2}{4}=0,$$

$$b_0\bar{h}^R+\int_{b_0}^{b^R}\frac{y+\ln a_0-u-x(t+\tau)-\eta\left(\bar{h}^R-\bar{h}_0\right)}{2}dx=n.$$

Solving these, we have the equilibrium solutions as:

$$b^R\left(\bar{h}^R\right)=b_0+2\sqrt{\frac{n-b_0\bar{h}^R}{t+\tau}}, \tag{13.1}$$

$$u^R\left(\bar{h}^R\right)=y+\ln a_0-b^R\left(\bar{h}^R\right)(t+\tau)-\eta\left(\bar{h}^R-\bar{h}_0\right). \tag{13.2}$$

From Eqs. (13), we have Lemma 1, which shows the effect of the ideal building height on the equilibrium solutions:

Lemma 1

The higher regulated building height leads to the smaller city and the higher utility.

It is quite straightforward to obtain by differentiating Eqs. (13) with respect to $\bar{h}^R$. Since the higher ideal building height results in the larger population within the historical district, the urban boundary shifts inwards. Consequently, the more compact city leads to the lower commuting cost; therefore, residents can enjoy the higher utility. In addition, substituting Eq. (13.1) into (11.2), the building height regulation is always valid for the entire historical district if:

$$\lambda^R\left(b_0,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=(t+\tau)\left(b^R\left(\bar{h}^R\right)-b_0\right)-2\bar{h}^R>0$$
$$\leftrightarrow \bar{h}^R<\tilde{h}=\frac{\sqrt{(t+\tau)\left\{4n+b_0^{\,2}(t+\tau)\right\}}-b_0(t+\tau)}{2}. \tag{14}$$

Substituting Eqs. (13) into (11) and (12), the equilibrium land rent and investment level are computed as:

$$S^R\left(x,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=\begin{cases}\bar{h}^{R2} & \text{for } 0\le x\le b_0,\\ \dfrac{(t+\tau)^2\left(b^R\left(\bar{h}^R\right)-x\right)^2}{4} & \text{for } b_0<x,\end{cases} \tag{15.1}$$

$$r^R\left(x,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=\begin{cases}\bar{h}^R(t+\tau)\left(b^R\left(\bar{h}^R\right)-x\right)-\bar{h}_0^{\,2} & \text{for } 0\le x\le b_0,\\ \dfrac{(t+\tau)^2\left(b^R\left(\bar{h}^R\right)-x\right)^2}{4} & \text{for } b_0<x.\end{cases}$$

(15.2)

By using these, we have Lemma 2:

Lemma 2

At $x=b_0$, the land rent within the district is lower than the one outside the district.

Proof: Let us respectively denote by b_0^- and b_0^+ the locations just inside and outside the boundary of the district. By comparing the land rents,

$$r^R\left(b_0^-,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=\bar{h}^R\left\{(t+\tau)b^R\left(\bar{h}^R\right)-\bar{h}^R\right\},$$

$$r^R\left(b_0^+,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=\frac{(t+\tau)^2\left(b^R\left(\bar{h}^R\right)-b_0^+\right)^2}{4}.$$

Since $\bar{h}^R<S^R\left(b_0,\bar{h}^R,u^R(\bar{h}^R)\right)$, we have $r^R\left(b_0^-,\bar{h}^R,u^R(\bar{h}^R)\right)<r^R\left(b_0^+,\bar{h}^R,u^R(\bar{h}^R)\right)$.

QED

According to Lemma 2, under the building regulation, the equilibrium land rent jumps at $x=b_0$. Since within the district, developers face the building height regulation, they cannot maximize the profit; consequently, the land rent within the district is lowered compared to the case without the regulation. In contrast to the land rent, the equilibrium housing rent, $p^R\left(x, \bar{h}^R, u^R(\bar{h}^R)\right)$ is continuous in the distance to the CBD: namely,

$$p^R\left(x, \bar{h}^R, u^R\left(\bar{h}^R\right)\right) = (t+\tau)\left(b^R\left(\bar{h}^R\right) - x\right). \tag{15.3}$$

3.2. The Equilibrium without the Building Height Regulation

As explained above, we focus on the case where the government abolishes the building height regulation. In addition, when deregulating, the government introduces the compensation, s, to all residents in order to assure that each resident can enjoy the utility, $u^R(\bar{h}^R)$.[1] The compensation may take either positive or negative sign. Once deregulated, the residents' welfare is harmed if the positive compensation is introduced while it is improved if the compensation takes the negative sign. In other words, in case of the negative compensation, residents are willing to pay for abolishing the building height regulation.

Since the no regulation is introduced, each developer x solves the problem (9). When solving the problem (9), each developer within the district does not consider the effect of their choices on the amenity value. Under this situation, the solution is computed as:

$$S^C\left(x, \bar{h}^C, u^R\left(\bar{h}^R\right)\right) = \frac{\left\{y + s + \ln a_0 - u^R\left(\bar{h}^R\right) - x(t+\tau) - \eta\left(\bar{h}^C - \bar{h}_0\right)\right\}^2}{4},$$

[1] Since we have assumed the quasi linear utility function, the compensation s just measures the difference in the utility in monetary term (or the difference in the composite good consumption) between the two alternative regimes. In other words, we can evaluate the welfare by aggregating the residents' indirect utility and the equilibrium land rent. Once apart from the quasi linear utility function, however, in order to measure the economic welfare in monetary term, we need to introduce the compensation.

(16.1)

$$\bar{h}^C\left(u^R\left(\bar{h}^R\right),s\right)=\frac{2\left(s+\eta\bar{h}^R\right)+\left(t+\tau\right)\left(2b^R\left(\bar{h}^R\right)-b_0\right)}{4+2\eta}. \quad (16.2)$$

Substituting Eqs. (16) into (7) and using the zero-profit condition, we have the land rent as:

$$r^C\left(x,\bar{h}^C,u^R\left(\bar{h}^R\right)\right)=\frac{\left\{y+s+\ln a_0-u^R\left(\bar{h}^R\right)-x\left(t+\tau\right)-\eta\left(\bar{h}^C-\bar{h}_0\right)\right\}^2}{4}$$

(17)

According Eqs. (10), (16), and (17), the equilibrium conditions are characterized as:

$$r^C\left(b^C,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)=\frac{\left\{y+s+\ln a_0-u^R\left(\bar{h}^R\right)-b^C\left(t+\tau\right)-\eta\left(\bar{h}^C-\bar{h}_0\right)\right\}^2}{4}=0,$$

(18.1)

$$\int_0^{b^C}\frac{y+s+\ln a_0-u^R\left(\bar{h}^R\right)-x\left(t+\tau\right)-\eta\left(\bar{h}^C-\bar{h}_0\right)}{2}dx=n. \quad (18.2)$$

Solving these, the equilibrium solutions, b^C and s^C, are computed as:

$$b^C=\sqrt{\frac{4n}{t+\tau}}, \quad (19.1)$$

$$s^C\left(\bar{h}^R\right)=\left(t+\tau\right)\left(b^C-b^R\left(\bar{h}^R\right)\right)+\eta\left(\left|\bar{h}^C-\bar{h}_0\right|-\left|\bar{h}^R-\bar{h}_0\right|\right), \quad (19.2)$$

where

$$\bar{h}^C=\frac{\left(t+\tau\right)\left(2b^C-b_0\right)}{4}. \quad (19.3)$$

Eq. (19.2) indicates the residents' tradeoff: namely, the first term of the RHS measures the gain from the compact city while the second term captures the loss due to the overinvestment within the district.[2] If $s^C(\bar{h}_0) > 0$, the residents receive the gain from the regulation since the gain from the regulation dominates the loss due to the expansion of the city. If $s^C(\bar{h}_0) < 0$, the residents strictly prefer the deregulation since the reduction in the commuting cost dominates the loss in the amenity value.

Eq. (19.3) shows the average building height under the laissez faire development. Lemma 3 summarizes the comparative statics on the average height, $\bar{h}^C$, with respect to the access cost parameters, t and τ.

Lemma 3

The marginal increase in the access costs to the city center, t and τ, augment the average building height, $\bar{h}^C$.

Proof: According to Eq. (19.3),

$$\frac{\partial \bar{h}^C}{\partial t} = \frac{\partial \bar{h}^C}{\partial \tau} = \frac{2b^C - b_0}{4} + \frac{t+\tau}{2}\frac{\partial b^C}{\partial t} = \frac{b^C - b_0}{4} > 0.$$

QED

Lemma 3 shows that, under the laissez faire development, the developers invest more to the historical district as the access condition to the city center is worsened. This is because

[2] If we allow that the residents attain the different utility level, u^C, from the one under the regulation, u^R, according to the equilibrium conditions, it is computed as:

$$u^C = y + \ln a_0 - b^C(t+\tau) - \eta(\bar{h}^C - \bar{h}_0).$$

By taking the difference, $u^R - u^C$, we have the compensation computed in Eq. (18.2). In other words, $u^R > u^C$ if $s^C > 0$ while $u^R < u^C$ if $s^C < 0$.

the worsened access makes the locations outside the district less attractive to the residents; therefore, under this circumstance, the developers have the stronger incentives to invest within the district. Pplugging Eqs. (19) into Eqs. (16) and (17), under the laissez faire development, the equilibrium land rent and investment level are derived as:

$$r^C\left(x,\bar{h}^C,u^R\left(\bar{h}^R\right)\right)=S^C\left(x,\bar{h}^C,u^R\left(\bar{h}^R\right)\right)=\frac{\left(t+\tau\right)^2\left(b^C-x\right)^2}{4}. \quad (20.1)$$

According to this, in contrast to the equilibrium with the building height regulation, the land rent is continuous at $x=b_0$. Finally, the equilibrium housing rent, $p^C\left(x,\bar{h}^C,u^R(\bar{h}^R)\right)$ is computed as:

$$p^C\left(x,\bar{h}^C,u^R\left(\bar{h}^R\right)\right)=\left(t+\tau\right)\left(b^C-x\right). \quad (20.2)$$

3.3. Comparison of Equilibrium Outcomes

This subsection compares the equilibrium outcomes under the two alternative regimes. We start with the effect of the access cost on the urban boundaries under the two alternative situations.

Lemma 4

As the access cost to the city center, t *or* τ*, rises, the urban boundaries shrink; furthermore, the boundary under the laissez faire development,* b^C*, shrinks more rapidly than the one under the regulation,* $b^R(\bar{h}^R)$*.*

Proof: Differentiating $b^R(\bar{h}^R)$ and b^C with respect to t or τ, we have:

$$\frac{\partial b^R\left(\bar{h}^R\right)}{\partial t}=\frac{\partial b^R\left(\bar{h}^R\right)}{\partial \tau}=-\sqrt{\frac{n-b_0\bar{h}^R}{\left(t+\tau\right)^3}}>-\sqrt{\frac{n}{\left(t+\tau\right)^3}}=\frac{\partial b^C}{\partial \tau}=\frac{\partial b^C}{\partial t}.$$

QED

Since the marginal increase in the access cost makes the locations in the suburb less attractive, the developers have stronger incentives to construct the higher buildings within the city. Under the regulation, however, the developers cannot construct the buildings higher than the regulated height, $\bar{h}^R$; therefore, the degree of the shrinkage is mitigated compared to the laissez faire.

The relation between the regulated and the average heights, ($\bar{h}^R$ and $\bar{h}^C$), affects the relation between the urban boundaries, $b^R(\bar{h}^R)$ and b^C. It is, however, natural to assume that the regulated height is lower than the average building height under the laissez faire: namely, $\bar{h}^R < \bar{h}^C$. Furthermore, as in Eq. (14), the regulation is effective if $\bar{h}^R < \tilde{h}$. By comparing these two thresholds,

$$\tilde{h}<\bar{h}^C \leftrightarrow b_0>\frac{8}{3}\sqrt{\frac{n}{t+\tau}}=\frac{4b^C}{3}>b^C.$$

Since $b_0 < b^C$, $\bar{h}^C > \tilde{h}$. Hence, we focus on the case where $\bar{h}^R < \tilde{h} < \bar{h}^C$. Under this circumstance, we can draw the conclusion such that the building height regulation expands the urban boundary compared to the laissez faire. This is summarized in Lemma 5.

Lemma 5

If all developers in the historical district follow the building height regulation, the regulation results in the spatial expansion of the city compared to the laissez faire development: that is, $b^R(\bar{h}^R) > b^C$ *if* $\bar{h}^R < \tilde{h}$.

Proof: By comparing the urban boundaries, we have:

$$b^R\left(\bar{h}^R\right) - b^C = b_0 + 2\sqrt{\frac{n - b_0\bar{h}^R}{t+\tau}} - \sqrt{\frac{4n}{t+\tau}}.$$

The urban boundary under the regulation coincides with the one under the laissez faire development (that is, $b^R(\bar{h}^R) = b^C$) if:

$$b^R\left(\bar{h}^R\right) - b^C = 0 \leftrightarrow \bar{h}^R = \frac{(t+\tau)\left(2b^C - b_0\right)}{4} = \bar{h}^C.$$

Since $b^{R\prime}(\bar{h}^R) < 0$, for $\bar{h}^R < \tilde{h} < \bar{h}^C$, $b^R(\bar{h}^R) > b^C$.

QED

Lemma 5 shows that the regulation makes the city spatially larger than the laissez faire development does. Since the regulation results in lower buildings within the district, in order to accommodate all households in the city, it is necessary to construct the more residence in the suburb. This implies that, according to Eqs. (15.3) and (20.2), the regulation augments the housing rent compared to the laissez faire development. Consequently, outside the district, the regulation leads to the higher land rent through the comparison of Eqs. (15.2) and (20.1). Namely, since $b^R(\bar{h}^R) > b^C$, for $b_0 \leq x$,

$$r^R\left(x, \bar{h}^R, u^R\left(\bar{h}^R\right)\right) = \frac{(t+\tau)^2\left(b^R\left(\bar{h}^R\right) - x\right)^2}{4} > \frac{(t+\tau)^2\left(b^C - x\right)^2}{4} = r^C\left(x, \bar{h}^C, u^R\left(\bar{h}^R\right)\right).$$

(21)

This indicates that, for $b_0 \leq x$, the landlords enjoy the net gain under the building height regulation.

In contrast, inside the district ($0 \leq x < b_0$), it is ambiguous to evaluate the effect of the building height regulation on the land rent. By comparing the land rents under the two alternative situations, we have Lemma 6.

Lemma 6

Within the district, the building height regulation always lowers the land rent if the regulated height, $\bar{h}^R$*, is lower than* $\hat{h}$*; otherwise, the regulation lowers the land rent for* $0 \leq x \leq \hat{x}$ *while it raises the rent for* $\hat{x} < x \leq b_0$.

Proof: Within the district, taking the difference in the land rents,

$$r^R\left(x,\bar{h}^R,u^R\left(\bar{h}^R\right)\right)-r^C\left(x,\bar{h}^C,u^R\left(\bar{h}^R\right)\right)=\bar{h}^R\left(t+\tau\right)\left(b^R\left(\bar{h}^R\right)-x\right)-\bar{h}^{R2}-\frac{\left(t+\tau\right)^2\left(b^C-x\right)^2}{4}.$$

Solving this for x, we have:

$$x=b^C-\frac{2\bar{h}^R}{t+\tau}\pm\sqrt{\frac{4\bar{h}^R\left(b^R\left(\bar{h}^R\right)-b^C\right)}{t+\tau}}.$$

Since we focus on the case where $x \leq b_0 < b^C$, we define $\hat{x}$ as:

$$\hat{x}=b^C-\frac{2\bar{h}^R}{t+\tau}-\sqrt{\frac{4\bar{h}^R\left(b^R\left(\bar{h}^R\right)-b^C\right)}{t+\tau}}.$$

For $x \leq \hat{x}$, $r^R\left(x, \bar{h}^R, u^R(\bar{h}^R)\right) < r^C\left(x, \bar{h}^C, u^R(\bar{h}^R)\right)$. At $\bar{h}^R = 0$, $\hat{x} = b^C > b_0$; therefore, in such case, $r^R(x, 0, u^R(0)) < r^C\left(x, \bar{h}^C, u^R(0)\right)$ holds for $0 \leq x \leq b_0$. Through the calculation, the threshold, $\hat{x}$, is decreasing in $\bar{h}^R$, $d\hat{x}/d\bar{h}^R < 0$, for $\bar{h}^R < \tilde{h}$. In other words, we can compute a threshold $\bar{h}^R = \hat{h}$ according to $\hat{x} = b_0$. If $\bar{h}^R \leq \hat{h}$, within the district, $r^R\left(x, \bar{h}^R, u^R(\bar{h}^R)\right) < r^C\left(x, \bar{h}^C, u^R(\bar{h}^R)\right)$ while, if $\bar{h}^R > \hat{h}$, $r^R\left(x, \bar{h}^R, u^R(\bar{h}^R)\right) \leq r^C\left(x, \bar{h}^C, u^R(\bar{h}^R)\right)$ for $0 \leq x \leq \hat{x}$; for $\hat{x} < x \leq b_0$, $r^R\left(x, \bar{h}^R, u^R(\bar{h}^R)\right) > r^C\left(x, \bar{h}^C, u^R(\bar{h}^R)\right)$.

QED

Lemma 6 shows that, when the regulated height is sufficiently low, the building height regulation results in the lower land rent compared to the laissez faire development. In contrast, by comparing with the laissez faire development, if the regulated height is moderate, the regulation lowers the land rent around the city center while it augments the land rent at the edge of the historical district. This indicates that, inside the historical district, the regulation hinders developers' opportunities to construct the residence; consequently, the landlords around the city center experience the loss through the regulation.

4. Welfare Effects of the Regulation

In this section, we evaluate the welfare effect of the building height regulation. First, we deal with incidence of the net welfare gain of the regulation among the different agents such as residents, developers, and landlords. Afterwards, we derive the condition where the building height regulation generates the positive net welfare gain. This subsection also provides the condition where the building height regulation becomes the second best. In this model, although we have the three types of agents, due to the competition, the developer's surplus is fully transferred to landlords.[3] Namely, we can limit our focus to how the building height regulation affects the welfares of residents and landlords.

We start with how the building height regulation affects the landlords' welfare, the aggregate land rent. According to Eqs. (15.2) and (20.1), the aggregate land rents are computed as:

$$R^R\left(\bar{h}^R\right)=\frac{(t+\tau)^2\left(b^R\left(\bar{h}^R\right)-b_0\right)^3}{12}+\frac{\bar{h}^R(t+\tau)\left\{b^R\left(\bar{h}^R\right)^2-\left(b^R\left(\bar{h}^R\right)-b_0\right)^2\right\}}{2}-b_0\bar{h}^{R2},$$

(22.1)

$$R^C=\frac{nb^C(t+\tau)}{3}=\frac{(t+\tau)^2\left(b^C-b_0\right)^3}{12}+\frac{(t+\tau)^2\left\{b^{C3}-\left(b^C-b_0\right)^3\right\}}{12}.$$

(22.2)

In the RHS of Eqs. (22), the first terms capture the aggregate land rent for the outside of

[3] In order to attain the opportunity to provide housing service at a location x, each developer must offer to a landlord the maximal willingness to pay; otherwise, they lose the opportunity to develop by their potential competitors.

the district while the last terms represent the one for the inside of the district.

As in Lemma 6, for the outside of the district, since the building height regulation augments the land rent, the landlords are better off through the regulation. For the inside of the district, the regulation lowers the rent; as a result, some of the landlords, especially those who own the land close to the CBD, are worse off. In aggregate, however, it is unclear to tell whether the regulation improves the landlords' welfare. Proposition 1 shows that the gain in the outside of the district always dominates the loss in the inside.

Proposition 1

If all developers within the historical district follows the regulated height, $0 < \bar{h}^R < \tilde{h}$, *the building height regulation always augments the aggregate land rent.*

Proof: When $\bar{h}^R = 0$, $b^R(0) - b_0 = b^C$; therefore, according to Eqs. (22), $R^R(0) = R^C$. In addition, differentiating Eq. (22.1) with respect to $\bar{h}^R$, we have:

$$R^{R\prime}\left(\bar{h}^R\right) = \frac{b_0}{2}\left\{b^R\left(\bar{h}^R\right)(t+\tau) - \frac{4b^R\left(\bar{h}^R\right)\bar{h}^R}{b^R\left(\bar{h}^R\right) - b_0}\right\}.$$

At $\bar{h}^R = 0$, $R^{R\prime}(0) > 0$. Furthermore, through the calculation, $R^{R\prime}\left(\bar{h}^R\right) \geq 0$ for $\bar{h}^R \leq \underline{h}$ where

$$\underline{h} = \frac{\sqrt{(t+\tau)\left\{16n + b_0^2(t+\tau)\right\}} - b_0(t+\tau)}{8}.$$

Since $\underline{h} < \tilde{h}$, $R^R\left(\bar{h}^R\right) > R^C$ for $0 < \bar{h}^R \leq \underline{h}$.

For $\underline{h} < \bar{h}^R < \tilde{h}$, $R^{R'}(\bar{h}^R) < 0$; hence, the building height regulation results in the larger aggregate land rent if $R^R(\tilde{h}) > R^C$. Since $\tilde{h} < \bar{h}^C$, $b^R(\tilde{h}) > b^C$; namely, for the outside of the district, the aggregate land rent under the regulation is always larger than the one under the laissez faire development. Therefore, we can limit our focus to the inside of the district. Let us respectively denote by $R_I^R(\bar{h}^R)$ and R_I^C the aggregate land rents under the regulation and the laissez faire. They are computed as:

$$R_I^R\left(\bar{h}^R\right) = \frac{b_0\bar{h}^R(t+\tau)\left(2b^R\left(\bar{h}^R\right)-b_0\right)}{2} - b_0\bar{h}^{R2},$$

$$R_I^C = \frac{(t+\tau)^2\left\{b^{C3}-\left(b^C-b_0\right)^3\right\}}{12} = b_0\bar{h}^{C2} + \frac{b_0{}^3(t+\tau)^2}{48}.$$

Since $b_0 < b^C$, by using $0 < \alpha < 1$, b_0 is rewritten as:

$$b_0 = \alpha b^C = 2\alpha\sqrt{\frac{n}{t+\tau}}.$$

According to this, $\tilde{h}$, $\bar{h}^C$, and the aggregate land rent differential at $\bar{h}^R = \tilde{h}$are expressed as functions of α:

$$\tilde{h} = \frac{b^C(t+\tau)\left(\sqrt{1+\alpha^2}-\alpha\right)}{2},$$

$$\bar{h}^C = \frac{b^C(t+\tau)(2-\alpha)}{4},$$

$$R_I^R\left(\tilde{h}\right) - R_I^C = \frac{nb^C F(\alpha)(t+\tau)}{3},$$

where

$$F(\alpha) = \alpha\left\{\alpha\left(3-10\alpha+9\sqrt{1+\alpha^2}\right)+6\sqrt{1-2\alpha\left(\sqrt{1+\alpha^2}-\alpha\right)}\left(\sqrt{1+\alpha^2}-\alpha\right)-\alpha\right\}$$

Through the calculation, we can easily confirm that $F(\alpha)$ is monotonically increasing for $0 < \alpha < 1$, and $F(0) = 0$. In sum, since $R^{R'}(\bar{h}^R) < 0$ for $\underline{h} < \bar{h}^R < \tilde{h}$ and $R^R(\tilde{h}) > R^C$, we can conclude that $R^R(\bar{h}^R) > R^C$ for $\underline{h} < \bar{h}^R < \tilde{h}$.

QED

Proposition 1 shows that, at the aggregate level, the landlords are better off through the building height regulation. For the outside of the historical district, the regulation augments the aggregate land rent through the spatial expansion of the urban boundary. For the inside of the district, since, as in Lemma 6, the regulation prevents the investment by developers, the regulation leads to the reduction in the aggregate land rent. In sum, however, Proposition 1 shows that the gain at the outside of the district dominates the loss at the inside district. In addition, as the regulated height rises, the gain at the outside shrinks since the city becomes more compact. The loss at the inside also shrinks as the regulation mitigates since the relaxation in the regulated height enhances the developers' investment. Hence, Proposition 1 also claims that, for any regulated height, the regulation improves the aggregate welfare of the landlords as long as the developers within the historical district follows the regulated height.

The net welfare gain of residents is measured by the compensation $s^C(\bar{h}^R)$. If $s^C(\bar{h}^R) < 0$, the residents strictly prefer the laissez faire development to the building height regulation since they are willing to pay to deregulate; otherwise ($s^C(\bar{h}^R) > 0$), they are better off through the regulation since they want to compensate if deregulated. As in Eq. (19.2), with respect to the disutility of the investment in the district, the compensation depends on the relation between the ideal and the regulated heights, $\bar{h}_0$ and $\bar{h}^R$. Namely, Eq. (19.2) is rewritten as:

$$s^C\left(\bar{h}^R\right) = \begin{cases} (t+\tau)\left(b^C - b^R\left(\bar{h}^R\right)\right) + \eta\left(\bar{h}^C - \bar{h}^R\right) \text{ if } \bar{h}^R \geq \bar{h}_0, \\ (t+\tau)\left(b^C - b^R\left(\bar{h}^R\right)\right) + \eta\left(\bar{h}^C + \bar{h}^R - 2\bar{h}_0\right) \text{ if } \bar{h}^R < \bar{h}_0. \end{cases} \quad (23)$$

As in Eq. (23), the marginal effect on the compensation, $s^C(\bar{h}^R)$, of the regulated height, $\bar{h}^R$, depends on the relation between the ideal and the regulated heights. Besides, the compensation, $s^C(\bar{h}^R)$, is convex in the regulated height. These are summarized in Lemma 7.

Lemma 7

The compensation $s^C(\bar{h}^R)$ has the following properties: i) it is increasing in $\bar{h}^R$ if $\bar{h}^R < \bar{h}_0$ or if $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$ and $\eta \leq -b^{R'}(\tilde{h})(t+\tau)$; ii) it is convex in the regulated height, $\bar{h}^R$.

Proof: It is easy to confirm the second property by deriving the second derivative of $s^C(\bar{h}^R)$ with respect to $\bar{h}^R$:

$$s^{C''}\left(\bar{h}^R\right) = \frac{4b_0^{\,2}}{(t+\tau)\left(b^R - b_0\right)^3} > 0.$$

For the first property, in case of $\bar{h}^R < \bar{h}_0$, provided that $b^{R'}(\bar{h}^R) < 0$, the sign of the marginal effect of the regulated height is derived as:

$$s^{C'}(\bar{h}^R) = \eta - b^{R'}(\bar{h}^R)(t + \tau) > 0.$$

For $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$, the marginal effect of the regulated height is computed as:

$$s^{C'}(\bar{h}^R) = -\eta - b^{R'}(\bar{h}^R)(t + \tau).$$

Hence, as long as $\eta \leq -b^{R'}(\tilde{h})(t + \tau)$, $s^{C'}(\bar{h}^R) > 0$ for $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$.

QED

The regulation affects the residents' welfare through the two channels such as the increase in the access cost to the city center and the disutility due to the investment in the historical district. Lemma 7 shows that, when the regulated height is lower than the ideal height, the compensation increases as the regulated height rises since a rise in $\bar{h}^R$ reduces the disutility of the overinvestment as well as the access cost to the city center. For $\bar{h}^R \geq \bar{h}_0$, the marginal increase in the regulated height causes the overinvestment in the district and the shrinkage of the urban boundary at the same time. Therefore, the sign of $s^{C'}(\bar{h}^R)$ depends on the magnitudes of the two effects. Furthermore, the convexity of the compensation implies that independent from the sign of $s^{C'}(\bar{h}^R)$, the marginal effect of the regulation on the net welfare gain increases as the regulated height rises.

By using Lemma 7, Proposition 2 summarizes the condition where the building height regulation improves the residents' welfare: that is, $s^C(\bar{h}^R) \geq 0$.

Proposition 2

The building height regulation always improves the residents' welfare if $\bar{h}_0 \leq \bar{h}^C/2 - b_0(t+\tau)/2\eta$ *and* $\eta \geq \tilde{\eta}$*. Also, the residents are better off through the regulation if the regulated height suffices the following relation.*

$$0 \leq \bar{h}^R \leq \bar{h} \text{ if } \bar{h}_0 \leq \frac{\bar{h}^C}{2} - \frac{b_0(t+\tau)}{2\eta} \text{ and } -b^{R\prime}(\tilde{h})(t+\tau) < \eta < \tilde{\eta},$$

(24.1)

$$\underline{h} \leq \bar{h}^R \leq \tilde{h} \text{ if } \bar{h}_0 > \frac{\bar{h}^C}{2} - \frac{b_0(t+\tau)}{2\eta} \text{ and } \tilde{\eta} \leq \eta, \tag{24.2}$$

$$\underline{h} \leq \bar{h}^R \leq \bar{h} \text{ if } \bar{h}_0 > \frac{\bar{h}^C}{2} - \frac{b_0(t+\tau)}{2\eta} \text{ and } -b^{R\prime}(\tilde{h})(t+\tau) < \eta < \tilde{\eta},$$

(24.3)

where

$$\underline{h} = 2\bar{h}_0 - \frac{\bar{h}^C(\eta+2)}{\eta} - \frac{b_0(4-\eta)(t+\tau)}{2\eta^2} + \frac{\sqrt{(t+\tau)\left[\left\{2\eta\sqrt{n} + b_0(\eta+2)\sqrt{t+\tau}\right\}^2 - 2b_0\eta\left\{b_0(\eta+4)(t+\tau) + 4\eta\bar{h}_0\right\}\right]}}{\eta^2},$$

(24.4)

$$\bar{h} = \bar{h}^C + \frac{2(t+\tau)(b^C - b_0)}{\eta} - \frac{4b_0(t+\tau)}{\eta^2}, \tag{24.5}$$

$$\tilde{\eta} = \frac{(t+\tau)\left(b^R(\tilde{h}) - b^C\right)}{\bar{h}^C - \tilde{h}}. \tag{24.6}$$

Proof: We start with the case of $\eta \leq -b^{R\prime}(\tilde{h})(t+\tau)$. In this case, $s^C(\bar{h}^R)$ is monotonically increasing in $\bar{h}^R$. For $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$, however, $s^C(\bar{h}^R) > 0$ if:

$$\eta > \frac{(t+\tau)\left(b^R\left(\tilde{h}\right)-b^C\right)}{\bar{h}^C-\tilde{h}} = \frac{4b_0\sqrt{t+\tau}}{2\sqrt{n}+2\sqrt{n-b_0\tilde{h}}-b_0\sqrt{t+\tau}} > \frac{b_0\sqrt{t+\tau}}{\sqrt{n-b_0\tilde{h}}} = -(t+\tau)b^{R\prime}\left(\tilde{h}\right).$$

This indicates that the residents are always worse off if $s^C(\bar{h}^R)$ is monotonically increasing in $\bar{h}^R$. In other words, the building height regulation may improve the residents' welfare if the compensation, $s^C(\bar{h}^R)$, is single-peaked.

For $0 < \bar{h}^R < \bar{h}_0$, the compensation $s^C(\bar{h}^R)$ is monotonically increasing and convex in $\bar{h}^R$. First if $s^C(0) \geq 0$, for $0 < \bar{h}^R < \bar{h}_0$, the regulation always improves the residents' welfare, and this is the case if:

$$s^C(0) \geq 0 \leftrightarrow \bar{h}_0 \leq \frac{\bar{h}^C}{2} - \frac{b_0(t+\tau)}{2\eta}.$$

When $s^C(0) < 0$, for $0 < \bar{h}^R < \bar{h}_0$, since $s^C(\bar{h}^R)$ is increasing and convex in $\bar{h}^R$ as in Lemma 7, solving $s^C(\bar{h}^R) \geq 0$ for $\bar{h}^R$, we have:

$$\begin{aligned}\bar{h}^R \geq\ & 2\bar{h}_0 - \frac{\bar{h}^C(\eta+2)}{\eta} - \frac{b_0(4-\eta)(t+\tau)}{2\eta^2} \\ & + \frac{\sqrt{(t+\tau)\left[\left\{2\eta\sqrt{n}+b_0(\eta+2)\sqrt{t+\tau}\right\}^2 - 2b_0\eta\left\{b_0(\eta+4)(t+\tau)+4\eta\bar{h}_0\right\}\right]}}{\eta^2} = \underline{h}.\end{aligned}$$

Also, at $\bar{h}_0 = \bar{h}^C/2 - b_0(t+\tau)/2\eta$, $\underline{h} = 0$.

For $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$, $s^C(\bar{h}^R)$ is decreasing and convex in $\bar{h}^R$. Hence, if $s^C(\tilde{h}) \geq 0$, $s^C(\bar{h}^R) > 0$ for $\bar{h}_0 \leq \bar{h}^R < \tilde{h}$. This occurs if:

$$s^C\left(\tilde{h}\right) \geq 0 \leftrightarrow \eta \geq \tilde{\eta} = \frac{(t+\tau)\left(b^R\left(\tilde{h}\right)-b^C\right)}{\bar{h}^C-\tilde{h}}.$$

When $-b^{R'}(\tilde{h})(t+\tau) < \eta < \tilde{\eta}$, solving $s^C(\bar{h}^R) \geq 0$ for $\bar{h}^R$, we have:

$$\bar{h}^R \leq \bar{h}^C + \frac{2(t+\tau)(b^C - b_0)}{\eta} - \frac{4b_0(t+\tau)}{\eta^2} = \bar{h}, \bar{h}^C \leq \bar{h}^R \text{ if } \eta \leq \frac{b_0}{b^C - b_0},$$

$$\bar{h}^R \leq \bar{h}^C, \bar{h} = \bar{h}^C + \frac{2(t+\tau)(b^C - b_0)}{\eta} - \frac{4b_0(t+\tau)}{\eta^2} \leq \bar{h}^R \text{ if } \eta > \frac{b_0}{b^C - b_0}.$$

Summarizing these discussions, we have Eqs. (24).

QED

Proposition 2 shows the condition where the residents are better off through the building height regulation. First, the building height regulation always improves the residents' welfare if the ideal height, $\bar{h}_0$, is sufficiently low and if the marginal disutility of the overinvestment, η, is sufficiently large. We start by focusing on the case of $\bar{h}_0 \leq \bar{h}^R$. The building height regulation mitigates the disutility due to the gap between the average and the ideal heights while it also increases the access cost to the city center through the spatial expansion of the city. Hence, as the marginal disutility, η, rises, the former effect, the mitigation of the overinvestment, dominates the latter, the spatial expansion. Hence, for $\tilde{\eta} \leq \eta$, even if the regulated height is higher than the ideal height, the residents strictly prefer the regulation to the laissez-faire.

For the case where the regulated height is lower than the ideal ($\bar{h}^R < \bar{h}_0$), Proposition 2 shows that if the ideal height, $\bar{h}_0$, is sufficiently low, the regulation always improves the residents' welfare. This is interpreted as follows: since the ideal height is sufficiently low, the gap between the regulated and the ideal heights is small. Namely, even if the regulated height is too low relative to the ideal height, the disutility due to the gap becomes negligible; hence, the residents strictly prefer the regulation. For the moderate values of the marginal utility and the ideal height, the residents strictly prefer the regulation. This is similar to the extreme case. Namely, for $\bar{h}^R < \bar{h}_0$, the disutility due to the gap between the average and the ideal heights becomes negligible as the regulated height rises; therefore, for the moderate value of $\bar{h}^R$, the residents strictly prefer the regulation. For $\bar{h}_0 \leq \bar{h}^R$, as the marginal disutility falls, the magnitude of the access cost becomes significant; hence, the residents request the laissez-faire development if the regulated height is sufficiently high.

4.2. The Building Height Regulation as the Second-best

As discussed in Subsection 4.1, the regulation always augments the aggregate land rent while the effect on the residents' welfare depends on the magnitudes of the marginal disutility and the access cost. To clarify the condition where the building height regulation attains the second-best, we define the net welfare gain of the regulation, $\Delta W(\bar{h}^R)$:

$$\Delta W\left(\bar{h}^R\right) = ns^C\left(\bar{h}^R\right) + R^R\left(\bar{h}^R\right) - R^C. \tag{25}$$

According to Eq. (25), the building height regulation is the second-best if $\Delta W(\bar{h}^R) \geq 0$. To make a comparison with Bertaud and Brueckner (2005), we first focus on the case of

$\eta = 0$ as in Proposition 3.

Proposition 3

If the disutility of the overinvestment does not exist, $\eta = 0$*, the building height regulation never attains the second-best.*

Proof: Evaluating Eq. (25) at $\eta = 0$, $\Delta W(\bar{h}^R)\big|_{\eta=0}$, the first and the second derivatives with respect to $\bar{h}^R$ are computed as:

$$\Delta W'(\bar{h}^R)\Big|_{\eta=0} = \frac{b_0(t+\tau)\left(2b^R(\bar{h}^R) - b^C\right)}{2} - 2b_0\bar{h}^R,$$

$$\Delta W''(\bar{h}^R)\Big|_{\eta=0} = -2b_0\left(1 + \frac{b_0}{b^R(\bar{h}^R) - b_0}\right) < 0.$$

That is, $\Delta W(\bar{h}^R)\big|_{\eta=0}$ is concave in $\bar{h}^R$. Provided that $0 < \bar{h}^R < \tilde{h} < \bar{h}^C$, $\Delta W'(\bar{h}^R)\big|_{\eta=0} > 0$ for $0 < \bar{h}^R < \tilde{h}$ since $\Delta W'(\bar{h}^C)\big|_{\eta=0} = 0$. Besides, $\Delta W(\bar{h}^R)\big|_{\eta=0}$ attains the maximum at $\bar{h}^R = \bar{h}^C$, and its value is computed as:

$$\Delta W(\bar{h}^C)\Big|_{\eta=0} = -\frac{b_0^3(t+\tau)^2}{48} < 0.$$

This immediately indicates that if the overinvestment generates no disutility to the residents ($\eta = 0$), for $0 < \bar{h}^R < \tilde{h}$, the net welfare gain of the regulation is always negative ($\Delta W(\bar{h}^R)\big|_{\eta=0} < 0$).

QED

In Bertaud and Brueckner (2005), the building height regulation always worsens the residents' welfare through the increase in the access cost while its effect on the aggregate land rent is indeterminate. Therefore, it is ambiguous to tell whether the building height regulation improves the economic welfare compared to the laissez-faire development. Proposition 3, however, shows that, for the entire economy, the building height regulation harms the economic welfare if the marginal disutility of the overinvestment is absent ($\eta = 0$). When the marginal disutility is present ($\eta > 0$), as in Proposition 2, the residents may prefer the building height regulation if both the regulated height, $\bar{h}^R$, and the marginal disutility, η, are moderate or if η is sufficiently high. Hence, in such situation, we can conclude that the building height regulation attains the second-best since the regulation always augments the aggregate land rent as in Proposition 1.

<<Table 1: ABOUT HERE>>

In our setup, however, since the marginal disutility does not affect the equilibrium land rent, we cannot quantitatively evaluate its impact through the land rent observation. It is necessary to conduct the numerical simulation to quantify the size of the disutility relative to the aggregate land rent, at which the regulation becomes the second-best. The parameter values are adjusted so that the simulation results replicate the case of Kyoto metropolitan area in Japan. Specifically, we calibrate the parameters, the marginal disutility of the distance to the historical district, τ, and the marginal cost of developers,

c, so that the equilibrium urban boundary, $b^R(\bar{h}^R)$, and the land rent at the CBD, $r^R\left(0, \bar{h}^R, u(\bar{h}^R)\right)$ are equal to the observed data. Table 1 summarizes the parameter values. In the numerical simulation, we set the regulated height, $\bar{h}^R$, equal to the average regulated height within the central part of Kyoto. Since the lowest height is equal to 10 meters, through the simulation, we assume that the regulated height is higher than the ideal height: that is, $\bar{h}^R > \bar{h}_0$.

<<Figure 1: ABOUT HERE>>

<<Figure 2: ABOUT HERE>>

Based on these parameter values, Figures 1 and 2 respectively plot the land rents and the building heights under the two alternative regimes. In each figure, the bold line presents the outcome under the building height regulation while the dashed line shows the one under the laissez-faire development. As in Figure 1, within the district, the building height regulation lowers the land rent compared to laissez-faire development. Table 2 shows the equilibrium outcomes under the two alternative situations. According to Table 2, the building height regulation leads to the 48% lower average building height within the historical district than the laissez-faire development does. Consequently, under the regulation, the city expands, and the landlords experience the welfare gain.

<<Table 2: ABOUT HERE>>

Figure 3 plots the net welfare gain of the regulation, $\Delta W(\bar{h}^R)$, and the residents' net gain, $ns^C(\bar{h}^R)$, against the marginal disutility, η. In Figure 3, the black line shows the net welfare gain while the gray captures the residents' net gain. Since the residents' net gain is monotonically increasing in η, the net welfare gain increases as the marginal disutility, η, rises. As in Proposition 2, for $\eta \geq \tilde{\eta} = 19.56$, both the residents' net gain and the net welfare gain take the non-negative values. Furthermore, for $\tilde{\eta} > \eta \geq \underline{\eta} = 14.27$, although the regulation makes the residents worse off, the net welfare gain takes the non-negative value. At $\eta = \underline{\eta}$ and $\eta = \tilde{\eta}$, the gains from the building height regulation are computed as:

$$n\underline{\eta}\left(\bar{h}^C - \bar{h}^R\right) = 0.66 \times 10^9 \text{ and } n\tilde{\eta}\left(\bar{h}^C - \bar{h}^R\right) = 1.30 \times 10^9.$$

Compared to the aggregate land rent under the regulation, $R^R(\bar{h}^R)$, at $\eta = \underline{\eta}$, the gain from the regulation is equal to 11.65% of $R^R(\bar{h}^R)$ while, at $\eta = \tilde{\eta}$, it amounts to 23.00% of $R^R(\bar{h}^R)$. In other words, the regulation attains the second-best if the gain from the regulation, $n\eta(\bar{h}^C - \bar{h}^R)$, is larger than 11.65% of the aggregate land rent. Besides, the regulation makes the residents better off if the gain from the regulation is larger than a quarter of the aggregate land rent.

<<Figure 3: ABOUT HERE>>

Finally, Figure 4 plots the net welfare gain, $\Delta W(\bar{h}^R)$, against the regulated height, $\bar{h}^R$.

In Figure 4, the black line plots $\Delta W(\bar{h}^R)$ at $\eta = \underline{\eta}$ while the gray line shows the net welfare gain, $\Delta W(\bar{h}^R)$ at $\eta = \tilde{\eta}$. Figure 4 shows that $\Delta W(\bar{h}^R)$ is concave in the regulated height, $\bar{h}^R$ and single-peaked in $\bar{h}^R$. In other words, for each value of the marginal disutility, η, we can compute the welfare-maximizing regulated height. As the marginal disutility, η, rises, the welfare-maximizing regulated height rises. In summary, the regulation attains the second-best if the marginal disutility, η, is sufficiently high. In such situation, for each value of η, we can compute the welfare-maximizing regulated height.

<<Figure 4: ABOUT HERE>>

5. Conclusion

This study aims at evaluating the welfare effects of the building height regulation when the amenity is present. We have shown that the building height regulation results in the expansion of the city. Due to the expansion of the city, the regulation increases the aggregate land rent while the residents' net gain depends on the marginal disutility of the overinvestment. The regulation makes the residents better off if the marginal disutility is sufficiently high and the regulated height is moderate. Besides, the building height regulation attains the second-best if the residents are better off while the laissez-faire becomes the second-best if the marginal disutility of the overinvestment is negligible. Through the numerical simulation, it is shown that, in the case of Kyoto, Japan, the building height regulation is preferable if the disutility of the overinvestment is at least as large as approximately 10% of the aggregate land rent.

In reality, however, it is difficult to measure the marginal disutility on the amenity; therefore, it is necessary to empirically evaluate the effect of the amenity value on the land rent. Also note that, although we have assumed that the marginal utility of the amenity diminishes as the distance to the district increases, we need to clarify how the distance to a historical sight affects the utility of the amenity. Furthermore, we specify the amenity value as the function of the difference in the average and the ideal heights within the district and the distance to the historical site. We need to check whether this specification is appropriate through the empirical analysis.

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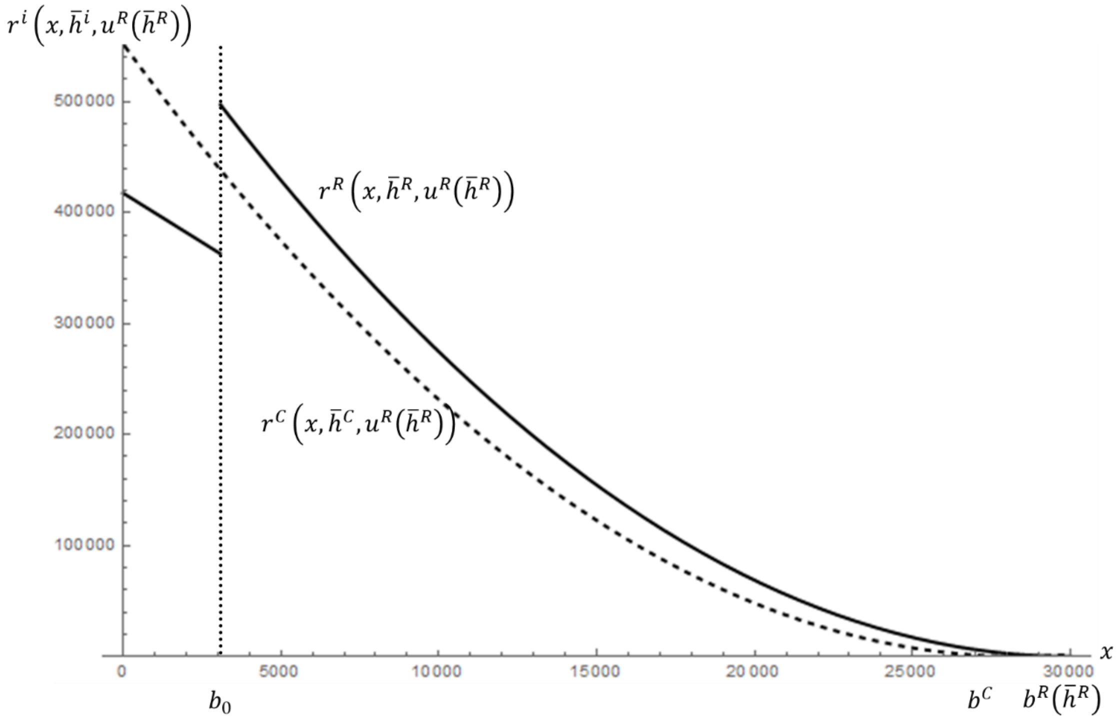

Figure 1: Equilibrium Land Rents under the Alternative Regimes

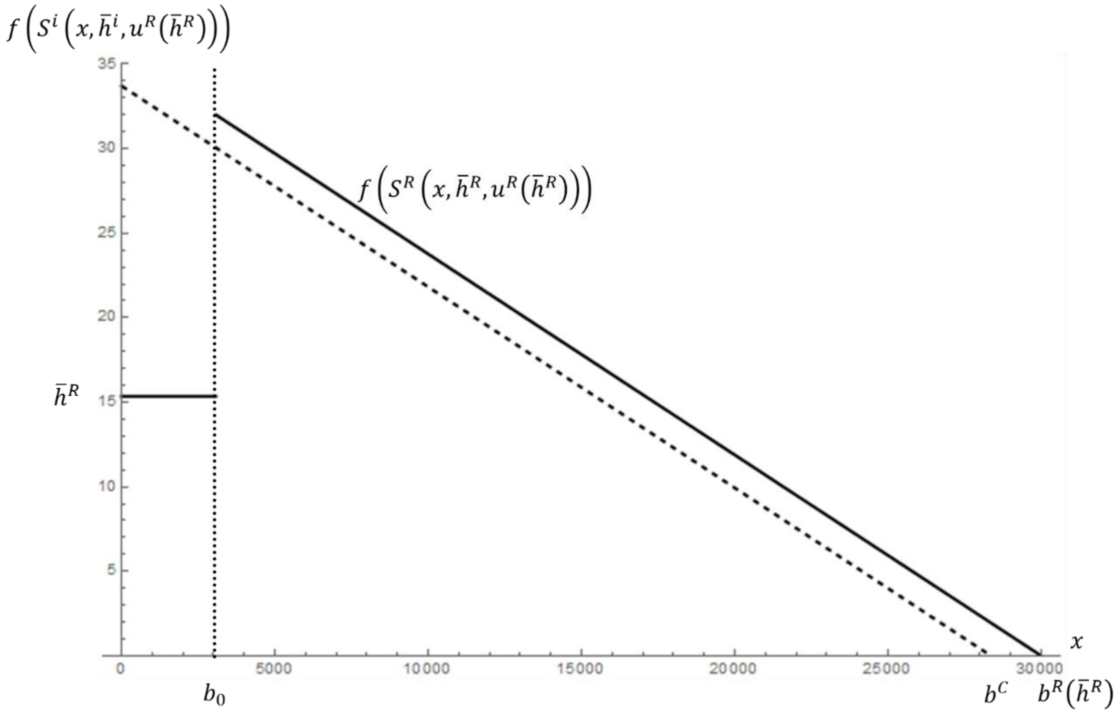

Figure 2: Equilibrium Building Heights under the Alternative Regimes

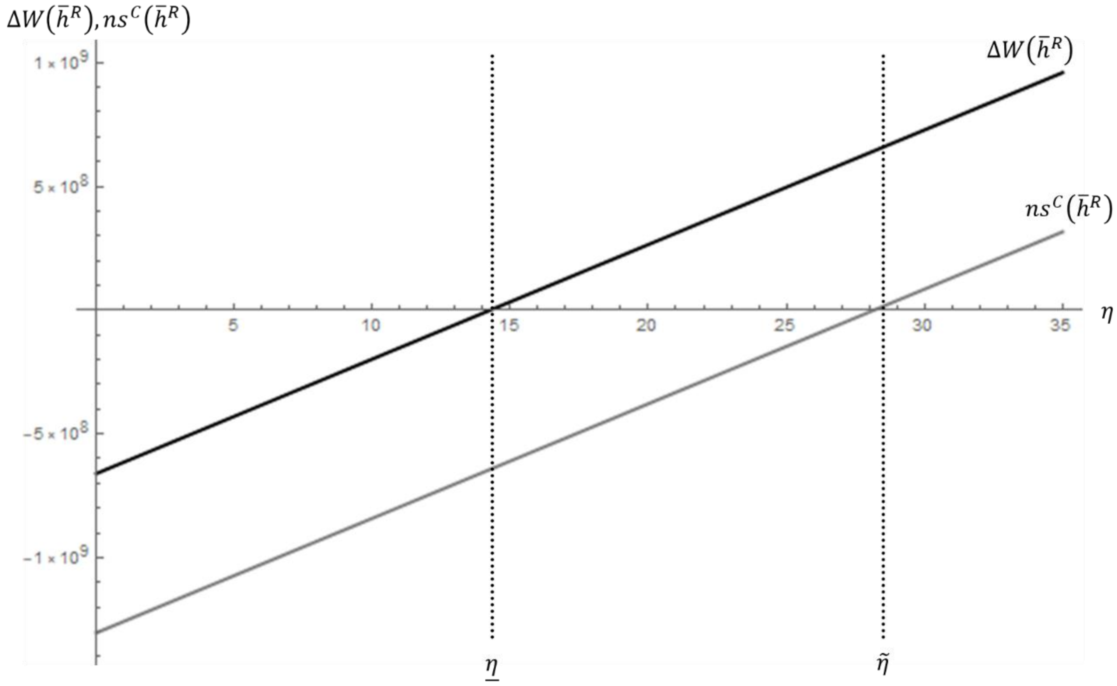

Figure 3: Effect of the Marginal Disutility on the Net Welfare Gains

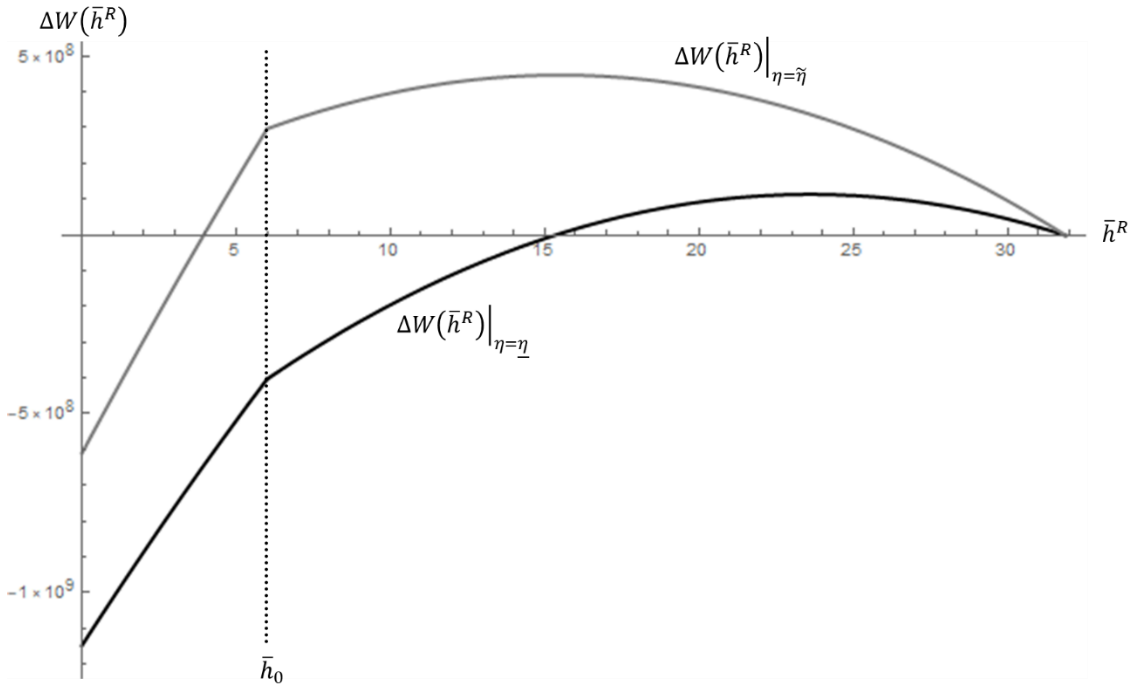

Figure 4: Effect of the Regulated Height on the Net Welfare Gain

Table 1: Summary of the Parameter Values

Parameter	Explanation	Value	Unit
n	Population of the city	2,081,000	Persons
t	Commuting cost	0.16	Yen/m
τ	Marginal disutility of the distance to the district	3.71×10^{-2}	Yen/m
$\bar{h}^R$	Regulated height	15.37	m
b_0	Size of the historical district	3078.58	m
q	Lot size	0.17	
c	Marginal cost of developers	485.78	Yen/m

Table 2: Comparison of the Equilibrium Outcomes

Building Height Regulation

Outcomes	Value	Unit
$\bar{h}^R$	15.37	m
$b^R(\bar{h}^R)$	30.00	km
$R^R(\bar{h}^R)$	8.16	billion yen

Laissez-faire Development

Outcomes	Value	Unit
$\bar{h}^C$	31.89	m
b^C	28.36	km
R^C	5.22	billion yen