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Universal Service Subsidy and its Effect on the Airline's Network Choice

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Abstract:
This paper constructs the three-airport model in which a full-service airline can determine its network configuration. Moreover, the population is heterogeneous in the willingness to pay and the size of each hinterland differs among the three airports. The airline faces the problem whether to provide the direct flight service to the thin demand routes. Furthermore, the government introduces the lump sum and the ad valorem subsidies to sustain the direct flight service to the thin demand route. By using this model, we address the following problems: i) how the subsidy affects the airline's choices on the airfare and the direct flight service; and ii) between the two alternative subsidies, which is the second-best. According to the comparison, we obtain the following results: i) the subsidy may enhance the air trip consumption if the proportion of travelers with high willingness to pay is large; and ii) with respect to the second-best policy, the ad valorem is more efficient if the population of travelers with high willingness to pay is large; the lump sum is more efficient, otherwise.

Keywords: Network Choice, Universal Service, Ad Valorem Subsidy, Lump Sum Subsidy

1. Introduction

Due to a significant decline in the population, Japanese local airports face the problem how to keep the passenger flight service. Indeed, from 2006 to 2015, although the Japanese airport users have increased by 8.47 %, the users of airports with less than 30 flights per day have declined by 7.47 %.[1] To sustain the direct flight service at local airports, the Japanese government implements the regulation on the flight service between the local airports and the Japanese largest hub, Tokyo International Airport (Hereafter, HND).[2] Among the regulated 19 local airports, significant declines in the travelers are

[1] Spitz et al. (2015) have reported that, from 2001 to 2013, the small airports in the United States have experienced 32 % decline in the flights, and 17 % decline in the available seats. Although, during the same period, the large hub airports have also faced the decreases in the flights and the available seats, the degree of the decline is less significant than the small airports experienced.
[2] Specifically, the regulation has the following two features: i) airlines cannot convert its slots for

reported in 11 airports. In addition to the regulation, the direct flights between HND and local airports receive the reduction in the landing fees. Furthermore, other than the favorable treatments at HND, the Japanese government subsidizes the airlines provide the service to the thin demand routes, especially to the remote islands. For the routes between remote islands and the mainland, the airlines receive the subsidy to the flight operating costs. In addition to the subsidy to the airline s' costs, since 2012, the Japanese government has started the subsidy to the airfare. Specifically, the subsidy takes the form of the ad valorem; the government subsidizes the difference in the predetermined base and the actual fares. This scheme aims at reducing the residents' access cost to the mainland level. In other words, with respect to the universal service of the free mobility, the subsidy may have a justification. On the other hand, since the thin demand route faces the lack in the competition, the service provider can exploit the consumer's surplus. Moreover, as in Valido et al. (2014), once the subsidy is introduced, the airline has a stronger incentive to raise the airfare. In other words, when introducing the subsidy, it is necessary to consider the tradeoff between the benefit of the remote island residents and the cost of the mainland residents.

Contrary to the remote island routes, at the national level, thin demand routes within

the thin demand routes to the thick demand routes; ii) once abolishing the routes to the thin demand airport, its slot is reallocated to other airlines.

the mainland have never receive the airfare subsidy since these routes have alternative mode of transportation. Indeed, among the routes under the regulation at HND, the local governments introduce several policies to enhance the air trip demand from the local airports such as the landing fee reduction and the lump sum subsidy to the residents. With respect to the free mobility, there exists a gap between residents in remote islands and rural regions in the mainland: namely, the remote island residents receive the subsidy at the national level while those in rural regions do not. Furthermore, the full service airlines such as All Nippon Airways and Japan Airlines claim that it is difficult to introduce the cross subsidy between the thick and the thin demand routes due to the population decline and the severe competition in the thick demand routes. Based on this background, we address the problem how the introduction of the subsidy affects the airline's choice on the direct flight service to thin demand routes.

This study constructs a model which is consisted from the three airports and evaluates the effects of the subsidy on the airline's choices on the airfare and the direct flight service. When introducing the subsidy, we consider two alternative schemes, the lump sum subsidy implemented in rural regions and the ad valorem subsidy implemented in remote islands. In addition, the subsidy is distributed to the residents in the small city. The population is heterogeneous in the willingness to pay and each resident makes a trip unless the trip cost exceeds the willingness to pay. Given the subsidy, the airline determines the choices on the airfare and the direct flight service to the smallest airport. Under this setup, we have the following results. First, under no subsidies, the airline's choice results in the undersupplies of the air trip service and of the direct flight service. Once the subsidy scheme is introduced, the undersupply of the air trip service is eliminated to some extent while the effect of the subsidy on the direct flight is ambiguous. In order to clarify this effect, we set the public expenditure of the subsidy identical between the two schemes. Under this setup, it is shown that the lump sum subsidy enhances the direct flight service if the proportion of the travelers with the high reservation price is low, or if this proportion and the transit cost incurred by travelers are high.

Several studies deal with the regulation at the congested hub and the airline's network choice. The studies on the regulation at the congested hub (for example, Brueckner 2009; Basso and Zhang 2010; and Sieg 2010) do not consider the difference in the market size among the routes explicitly. With respect to the competitive environment, Fukui (2010) addresses the problem whether the slot allocation enhances the competition among airlines. The studies on the airline's network choice (for example, Brueckner, 2004;

Kawasaki, 2008; Flores-Fillol, 2009)[3] do not focus on the effect of the policy on the airline's choice. Although the effect of the policy is studied in Matsumura and Matsushima (2010) and Teraji and Morimoto (2014), they consider the situation where all agents in the model are directly affected through the policy.[4]

With respect to the policy toward the thin demand routes, we have literatures such as Calzada and Fageda (2012), Valido et al. (2014), and Teraji and Araki (2016). Focusing on the Japanese regulation, Teraji and Araki (2016) compares the regulation on the slot and the discount in the landing fee. Although they show that both policies are equivalent in the efficiency perspective, they do not consider the transfer scheme among the passengers traveling different routes. Calzada and Fageda (2012) empirically evaluates the effect of the universal service obligation on the flight frequency and the airfare for the thin demand route. They show that the subsidy to the residents augments the airfare and the benefit of the subsidy is transferred to the airlines. Valido et al. (2014) also deals with the residence-based subsidy. They show that, as in Calzada and Fageda (2012), once the

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[3] Brueckner (2004) analyzes the topic using three airports and a monopolistic carrier model. The carrier chooses a hub-spoke network when the fixed cost for a flight is high relative to the marginal cost for a seat and when passengers place a high value on flight frequency. Kawasaki (2008) extends the model of Bruechner (2004) by introducing the heterogeneity in value of time among passengers, leisure and business demands. Flores-Fillol (2009) extends the model by considering the duopoly case and shows that asymmetric equilibria may arise, namely one carrier chooses a point-to-point network while the other chooses a hub-spoke network.

[4] Matsumura and Matsushima (2010) evaluates the welfare effects of the hub airport privatization while Teraji and Morimoto (2014) measures the effect of the fee at the hub on the airline's network choice.

subsidy is introduced, the airline receives the benefits of the subsidy through raising the airfare. This study extends the model of Valido et al. (2014) by introducing the airline's choice on the direct flight service.

This paper is organized as follows: Section 2 describes the model. In Section 3, we summarize the results of the social optimum and the equilibrium without subsidies. By comparing the results, we summarize how the airline distorts the airfare and the direct flight choices. In Section 4, we introduce the lump sum and the ad valorem subsidies and evaluates the effects of the two alternative subsidies on the airline's choices. Section 5 compares the effects of the subsidies and summarizes the condition where the two alternative subsidies become the second-best policy. Finally, Section 6 concludes.

2. The Model

Suppose an economy consisted from three cities. Each city i $(i = 1,2,S)$ has an airport, and we name the one located at i airport i. A single airline connects three airports by choosing the network configuration, hub-spoke and point-to-point. The distance between cities i and j is given by d_{ij}, and we normalize the distance between 1 and other two cities to unity: that is, $d_{12} = d_{1S} = 1$ and $d_{2S} = d < 2$. In addition, the population of city i is represented by n_i, and we set $n_1 = n_2 = 1$ and $n_S = n < 1$: namely, city S

is the smallest in the population size.

When choosing the network configuration, the airline always provides the direct flight service between city 1 and other two cities. Under this assumption, the network configuration depends on whether the airline provides the direct flight service between 2 and S. We denote by δ whether the airline chooses to provide direct flights route between 2 and S. Specifically, the airline serves the direct flight if $\delta = 1$ while no direct flight service if $\delta = 0$. With respect to the network configuration, the airline forms the point-to-point if $\delta = 1$; the network becomes the hub-spoke if $\delta = 0$. In other words, the airline faces the problem how to connect airport S to its network. Hereafter, in order to simplify the analysis, we omit the analysis of the airline's behavior for route between 1 and 2.

Residents in each city make trips unless the trip cost exceeds the reservation price. With respect to the reservation price, residents are divided into two types, b and l. The population ratio of type b is identical among three cities, we denote by α the population ratio of type b in each city. Without loss of generality, we assume the reservation price of type b, v^b, is higher than that of l, v^l. In addition, we normalize the type b's reservation price to unity ($v^b = 1$) and we denote by $v < 1$ the one for type l. The trip cost also differs between the two types. Specifically, type b travelers incur the transit cost, $1 - v > \mu > 0$, while type l travelers do not. Let us denote by $f_{iS}^t(\delta_{iS})$ the trip cost for type t $(t = b, l)$, and it is defined as:

$$f_{iS}^t(\delta_{iS}) = p_{iS} + \mu^t(1 - \delta_{iS}), \tag{1}$$

where p_{is} is the airfare for route between i and S and, by the assumption, $\mu^b = \mu > \mu^l = 0$. Furthermore, $\delta_{2S} = \delta$ while $\delta_{1S} = 1$ since the airline always provides the direct flight to the route between 1 and S.

Since we assume that type b's willingness to pay net of the transit cost is higher than type l's reservation price, the aggregate demand between i and S is computed as:

$$Q_{iS}(\mathbf{p}, \boldsymbol{\delta}) = \begin{cases} 1 + n \text{ if } p_{iS} \leq v, \\ \alpha(1+n) \text{ if } v < p_{iS} \leq 1 - \mu(1 - \delta_{iS}), \end{cases} \tag{2}$$

where $\mathbf{p} = (p_{1S}, p_{2S})$ and $\boldsymbol{\delta} = (\delta_{1S}, \delta_{2S}) = (1, \delta)$. In addition, each type's consumer surplus is:

$$u_{iS}^t(\mathbf{p}, \boldsymbol{\delta}) = \begin{cases} 0 \text{ if } p_{iS} \geq v^t - \mu(1 - \delta_{iS}), \\ v^t - p_{iS} \text{ if } p_{iS} < v^t - \mu(1 - \delta_{iS}). \end{cases} \tag{3}$$

The airline incurs the flight and the route operating costs. The flight operating cost is proportional to the passenger kilometer by c while the route operating cost is proportional to the number of the direct flight routes by F. In sum, the airline's total cost is computed as:

$$C(\mathbf{p}, \boldsymbol{\delta}) = c \sum_{i=1,2} Q_{iS}(\mathbf{p}, \boldsymbol{\delta}) \left\{ \delta_{iS} d_{iS} + (1 - \delta_{iS})(d_{jS} + 1) \right\} + F(1 + \delta_{2S}). \tag{4}$$

By using Eq. (4), the airline's profit is given by:

$$\pi(\mathbf{p},\boldsymbol{\delta}) = \sum_{i=1,2} p_{iS} Q_{iS}(\mathbf{p},\boldsymbol{\delta}) - C(\mathbf{p},\boldsymbol{\delta}).$$

According to this, the airline's problem is formulated as:

$$\max_{\mathbf{p},\delta_{2S}} \pi(\mathbf{p},\boldsymbol{\delta}). \tag{5}$$

Finally, using Eqs. (3) and (4), the social surplus is defined as:

$$SS(\mathbf{p},\boldsymbol{\delta}) = 2(1+n)\sum_{i=1,2}\left\{\alpha u_{iS}^{b}(p_{iS},\delta_{iS}) + u_{iS}^{l}(p_{iS})(1-\alpha)\right\} - C(\mathbf{p},\boldsymbol{\delta}). \tag{6}$$

At the optimum, the social surplus is maximized, and the optimal solutions are derived from the following problem:

$$\max_{\mathbf{p},\delta_{2S}} SS(\mathbf{p},\boldsymbol{\delta}). \tag{7}$$

We use the solutions of Eq. (7) as the benchmark to evaluate the welfare effects of the decentralized decision-making by the airline with and without the subsidy.

3. Social Optimum and Equilibrium

In order to evaluate how the subsidy affects the airline's choice, this section deals with the social optimum and the equilibrium without subsidy. Subsection 3.1 focuses on the social optimum while Subsection 3.2 addresses how, without subsidies, the airline's choice differs from the optimal one.

3.1. Social Optimum

The optimal solution is derived by solving the problem (7). We first focus on the optimal airfare with respect to whether assuring the consumption of air trip service by type l. In other words, the social planner chooses one of the two alternatives such as $p_{iS} \leq v$ or $p_{iS} > v$. The exclusion of type l is inefficient if:

$$v \geq c\left\{\delta_{2S} d + 2\left(1-\delta_{2S}\right)\right\}. \tag{8}$$

Once Eq. (8) is satisfied, the reservation price of type l always exceeds the flight operating cost. In other words, providing the service to l always generates the surplus. Hereafter, we consider the case where the optimal airfare, p_{iS}^{O}, is lower than the type l's willingness to pay.

Let us denote by $\mathbf{p^O} = (p_{1S}^{O}, p_{2S}^{O})$ the vector of the optimal airfares. The social surplus is computed as:

$$SS\left(\mathbf{p^O}, 1, \delta_{2S}\right) = \left(1+n\right)\left[2\left\{\alpha + v\left(1-\alpha\right)\right\} - \alpha\mu\left(1-\delta_{2S}\right) - c\left\{1+\delta_{2S} d + 2\left(1-\delta_{2S}\right)\right\}\right] - F\left(1+\delta_{2S}\right). \tag{9}$$

By comparing the social surplus under the two alternative situations, we have Lemma 1, which shows the optimal choice on the direct flight service between 2 and S:

Lemma 1

The optimal choice on the direct flight service between 2 *and* S *is characterized by:*

$$\delta_{2S}^{O} = \begin{cases} 0 \text{ if } F \geq F^{O} = (1+n)\{\alpha\mu + c(2-d)\}, \\ 1 \text{ if } F < F^{O} = (1+n)\{\alpha\mu + c(2-d)\}. \end{cases} \tag{10}$$

Proof: Evaluating Eq. (9) at $\delta_{2S} = 1$ and $\delta_{2S} = 0$, and solving the differential for F, we have F^{O}. According to the comparison of F^{O} and F, we have Eq. (10).

QED

The threshold, F^{O}, given by (10) shows the social planner's tradeoff on ceasing the direct flight service for the route between 2 and S. Namely, the right-hand side is the sum of type b's transit cost, $\alpha\mu(1+n)$, and the flight operating cost for the connecting flights, $c(2-d)(1+n)$ while the left-hand side is the route operating cost. In other words, Eq. (10) shows that the social planner chooses to operate the direct flight between 2 and S if the route operating cost is lower than the increase in the cost due to the transit service.

3.2. Equilibrium without Subsidies

The airline's choice is characterized by Eq. (5). The same as the social optimum, we start with the choice on the airfare, and then we consider the airline's network choice. When setting the airfare, the airline cannot distinguish the traveler's type prior to purchasing. Therefore, the airline faces the tradeoff setting the airfare equal to the type l's reservation price, v, or the type b's, $1-\mu(1-\delta_{iS})$. Under this tradeoff, the equilibrium airfare is derived from the following:

$$\mathbf{p}^*(\boldsymbol{\delta}) = \arg\max_{\mathbf{p}} \pi(\mathbf{p}, \boldsymbol{\delta}).$$

The result is summarized in Lemma 2:

Lemma 2

Given the airline's network choice, δ_{2S}*, the equilibrium airfare is derived as:*

$$p_{iS}^*(\boldsymbol{\delta}) = \begin{cases} v \text{ if } \alpha \le \alpha_{iS}^*(\delta_{iS}), \\ 1-\mu(1-\delta_{iS}) \text{ if } \alpha > \alpha_{iS}^*(\delta_{iS}). \end{cases} \quad (11.1)$$

where

$$\alpha_{iS}^*(\delta_{iS}) = \frac{v - c\left\{\delta_{iS} d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}}{1-\mu(1-\delta_{iS}) - c\left\{\delta_{iS} d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}} \text{ for } i = 1, 2,\ j \ne i.$$

(11.2)

Proof: When $p_{iS}^* = v$, as in Eq. (2), the air trip demand is equal to $1+n$; hence, the flight operating cost is equal to $c(1+n)\{\delta_{iS} l_{iS} + (1-\delta_{iS})(l_{jS}+1)\}$. In contrast, if $p_{iS}^* = 1-\mu(1-\delta_{iS})$, the demand is $\alpha(1+n)$ and the flight operating cost is $\alpha c(1+n)\{\delta_{iS} l_{iS} + (1-\delta_{iS})(l_{jS}+1)\}$. Taking the difference in the profits under the two alternatives, the airline prefers $p_{iS}^* = v$ if:

$$(1+n)\left[v - c\left\{\delta_{iS} d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\} - \alpha\left[1-\mu(1-\delta_{iS}) - c\left\{\delta_{iS} d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]\right] > 0.$$

Solving this for α, we have the threshold, $\alpha_{iS}^*(\delta_{iS})$, and we obtain Eqs. (11).

QED

The threshold, $\alpha_{iS}^*(\delta_{iS})$, captures the difference in the profit on per passenger basis between $p_{iS}^* = v$ and $p_{iS}^* = 1 - \mu(1 - \delta_{iS})$. Hence, if $\alpha > \alpha_{iS}^*(\delta_{iS})$, the airline chooses to exclude the air trip consumption by type l. In order to derive the airline's direct flight service choice between cities 2 and S, we compare $\alpha_{2S}^*(1)$ and $\alpha_{2S}^*(0)$:

$$\alpha_{2S}^*(0) - \alpha_{2S}^*(1) = \frac{\mu(v - cd) - c(1-v)(2-d)}{(1-\mu-2c)(1-cd)} > 0 \leftrightarrow \mu > \mu^* = \frac{c(1-v)(2-d)}{v - cd}. \tag{12}$$

Eq. (12) indicates that, if the transit cost is sufficiently large ($\mu > \mu^*$), it is easier for the airline to raise the airfare when providing the direct flight service between 2 and S. This is because, if the direct flight service is ceased, the gain from raising the airfare shrinks since type b incurs the transit cost; therefore, the airline abuses its market power when the direct flight service is provided.

Since we have assumed $\delta_{1S} = 1$, the equilibrium airfare given by (11.1), $\mathbf{p}^*(\boldsymbol{\delta})$, is written as the function of δ_{2S}: that is, $\mathbf{p}^*(\boldsymbol{\delta}) = \mathbf{p}^*(\delta_{2S})$. Substituting this into the airline's profit, the airline's choice on the direct flight, δ_{2S}^*, is obtained as:

$$\delta_{2S}^* = \arg\max_{\delta_{2S}} \pi\left(\mathbf{p}^*(\delta_{2S}), \delta_{2S}\right).$$

Lemma 3 summarizes the airline's choice on the direct flight service between 2 and S:

Lemma 3

When no subsidies are introduced, the equilibrium choice on the direct flight service *between* 2 *and* S *is characterized by:*

$$\delta_{2S}^{*}=\begin{cases}0 \text{ if } F \geq F^{*}(\alpha,\mu),\\ 1 \text{ if } F < F^{*}(\alpha,\mu),\end{cases} \tag{13.1}$$

where

$$F^{*}(\alpha,\mu)=\begin{cases} c(1+n)(2-d) \text{ if } 0<\alpha\leq\min\left\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\right\},\\ (1+n)\{\alpha-v-c(\alpha d-2)\} \text{ if } \alpha_{2S}^{*}(1)<\alpha\leq\alpha_{2S}^{*}(0) \text{ and } \mu>\mu^{*},\\ (1+n)\{v-\alpha(1-\mu)-c(d-2\alpha)\} \text{ if } \alpha_{2S}^{*}(0)<\alpha\leq\alpha_{2S}^{*}(1) \text{ and } \mu\leq\mu^{*},\\ \alpha(1+n)\{\mu+c(2-d)\} \text{ if } \max\left\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\right\}<\alpha<1.\end{cases}$$

(13.2)

Proof: According Eqs. (11) and (12), the airline's airfare choice is summarized as follows: first, independent from the choice on the direct flight service, $p_{2S}^{*}(\delta_{2S})=v$ if $\alpha\leq\min\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}$ while $p_{2S}^{*}(\delta_{2S})=1-\mu(1-\delta_{2S})$ if $\max\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}<\alpha$. In case of $\min\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}<\alpha\leq\max\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}$, the relation of the two thresholds depends on the size of the transit cost, μ. If $\mu\leq\mu^{*}$, $\min\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}=\alpha_{2S}^{*}(0)$; in this case, $p_{2S}^{*}(0)=1-\mu$ and $p_{2S}^{*}(1)=v$. In contrast, $\min\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}=\alpha_{2S}^{*}(1)$ if $\mu>\mu^{*}$: in this situation, $p_{2S}^{*}(1)=v$ and $p_{2S}^{*}(1)=1$. We start with the case of $\alpha\leq\min\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}$. In such case, the airline chooses $\delta_{2S}=1$ if

$$\pi(1)-\pi(0)=c(1+n)(2-d)-F>0\leftrightarrow F<c(1+n)(2-d),$$

while, for $\max\{\alpha_{2S}^{*}(0),\alpha_{2S}^{*}(1)\}<\alpha$, the airline prefers $\delta_{2S}=1$ if

$$\pi(1)-\pi(0)=\alpha(1+n)\{\mu+c(2-d)\}-F>0 \leftrightarrow F<\alpha(1+n)\{\mu+c(2-d)\}.$$

For $\mu \leq \mu^*$ and $\alpha_{2S}^*(0) \leq \alpha < \alpha_{2S}^*(1)$, $\delta_{2S} = 1$ emerges at the equilibrium if:

$$\pi(1)-\pi(0)=(1+n)\{v-\alpha(1-\mu)\}-c(1+n)(d-2\alpha)-F>0$$
$$\leftrightarrow F<(1+n)\{(v-cd)-\alpha(1-\mu-2c)\}.$$

Finally, for $\mu > \mu^*$ and $\alpha_{2S}^*(1) \leq \alpha < \alpha_{2S}^*(0)$,

$$\pi(1)-\pi(0)=(1+n)(\alpha-v)-c(1+n)(\alpha d-2)-F>0$$
$$\leftrightarrow F<(1+n)\{\alpha(1-cd)-(v-2c)\}.$$

Summarizing theses, we have Eqs. (13).

QED

Eq. (13.2) shows the airline's tradeoff on the direct flight service when no subsidies are introduced. Namely, when, independent from the direct flight choice, the airline serves to the same type, the airline determines its network configuration by comparing the route operating cost and the additional flight operating cost due to the connecting flights. In contrast, when the consumers of air trip service differ between the two networks, the airline chooses its network according to the comparison of the route operating cost and the change in the revenue as well as the increase in the flight operating cost.

By summarizing Lemmas 1, 2, and 3, we state Proposition 1, which summarizes the distortion due to the airline's choice under no subsidies:

Proposition 1

When no subsidies are introduced, if $\min\{\alpha_{1S}^*(1), \alpha_{2S}^*(0), \alpha_{2S}^*(1)\} < \alpha$, *the airline abuses its market power at least one of the two routes. Furthermore, even if the airline does not exercise its market power for the two routes, the airline ceases the direct flight service to route between* 2 *and* S *more easily than the social planner.*

Proof: At the optimum, $p_{iS}^O \leq v$ while, as in Lemma 2, for $\min\{\alpha_{1S}^*(1), \alpha_{2S}^*(0), \alpha_{2S}^*(1)\} < \alpha$, at least one of the two routes, we have $p_{iS}^O \leq v < p_{iS}^*(\delta_{iS}) = 1 - \mu(1 - \delta_{iS})$. For the second statement, by comparing Eqs. (13.2) and F^O, we have $F^O > F^*(\alpha, \mu)$.

QED

Proposition 1 shows that, without subsidy, the airline's decision generates two types of the inefficiency. First, due to the market power, by raising the airfare, the airline excludes the air trip service consumption by type l. In such situation, the airline ceases the direct flight service provision more easily than the efficient level. This is because, since, at the equilibrium, the air trip consumers are small, the airline can reduce the additional flight operating cost due to the transit. Furthermore, even if type l travelers are not excluded, at the equilibrium, the direct flight service is stopped more easily than at the optimum since the airline does not take type b's transit cost into account when determining the direct flight choice.

4. Equilibria under Lump Sum Subsidy and Ad Valorem Subsidy

As discussed in Section 3, when no subsidies are introduced, the airline's choice results in the exclusion of type l and the suboptimal provision of direct flight service between 2 and S. This section addresses the question how subsidies affect the airline's choice. When considering the subsidies, we introduce the two alternative systems such as lump sum and ad valorem subsidies. Given the subsidy, the airline determines the airfare and the choice on the direct flight service between 2 and S. Subsection 4.1 focuses on the case of the lump sum subsidy while Subsection 4.2 considers the ad valorem case.

4.1. Lump Sum Subsidy

Let us denote by T the lump sum subsidy per resident in city S. When the lump sum subsidy to residents in city S is introduced, with respect to the residence base, the difference in the reservation price appears. Namely, the willingness to pay of residents in S is higher than the one for residents of other cities by T. When assuming that the lump sum subsidy, T, is smaller than the difference in the reservation prices, $1-\mu-v$, the air trip demand between i and S is computed as:

$$Q_{iS}(\mathbf{p},T)=\begin{cases} 1+n \text{ if } 0 \le p_{iS} \le v, \\ 1+\alpha n \text{ if } v < p_{iS} \le v+T, \\ \alpha(1+n) \text{ if } v+T < p_{iS} \le 1-\mu(1-\delta_{iS}), \\ \alpha n \text{ if } 1-\mu(1-\delta_{iS}) < p_{iS} \le 1+T-\mu(1-\delta_{iS}). \end{cases} \tag{14}$$

By using Eq. (14), the airline's profit is computed as:

$$\pi(\mathbf{p},\boldsymbol{\delta},T)=\sum_{i=1,2}Q_{iS}(\mathbf{p},T)\left[p_{iS}-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]-F(1+\delta_{2S}).$$

Let us denote by $\mathbf{p}^{\mathbf{L}}(\boldsymbol{\delta},T)$ the equilibrium airfare under the lump sum subsidy. This is obtained as:

$$\mathbf{p}^{\mathbf{L}}(\boldsymbol{\delta},T)=\arg\max_{\mathbf{p}}\pi(\mathbf{p},\boldsymbol{\delta},T).$$

Lemma 4 shows the equilibrium airfare as follows:

Lemma 4

When residents in city S receive the lump sum subsidy, the equilibrium airfare, $\mathbf{p}^{\mathbf{L}}(\boldsymbol{\delta},T)$, is derived as:

$$p_{iS}^{L}(\delta_{iS},T)=\begin{cases} v \text{ if } \alpha\le\alpha_{iS}^{*}(\delta_{iS}) \text{ and } T\le\hat{T}_{iS}(\delta_{iS}), \\ v+T \text{ if } \max\left\{\hat{T}_{iS}(\delta_{iS}),\tilde{T}_{iS}(\delta_{iS})\right\}<T<1-\mu-v, \\ 1-\mu(1-\delta_{iS}) \text{ if } \alpha_{iS}^{*}(\delta_{iS})<\alpha \text{ and } T\le\tilde{T}_{iS}(\delta_{iS}). \end{cases}$$

(15.1)

where

$$\hat{T}_{iS}(\delta_{iS})=\frac{(1-\alpha)\left[v-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]}{\alpha+n}, \tag{15.2}$$

$$\tilde{T}_{iS}(\delta_{iS})=\frac{\alpha(1+n)\left\{1-\mu(1-\delta_{iS})-v\right\}-n(1-\alpha)\left[v-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]}{\alpha+n}.$$

(15.3)

Proof: As in Lemma 2, if $\alpha\le\alpha_{iS}^{*}(\delta_{iS})$, the airline strictly prefers $p_{iS}=v$ to $p_{iS}=1-$

$\mu(1-\delta_{iS})$ while, if $\alpha > \alpha_{iS}^{*}(\delta_{iS})$, the airline chooses $p_{iS} = 1-\mu(1-\delta_{iS})$. First, focusing on $T < 1-\mu-v$, by comparing the profits between $p_{iS} = 1-\mu(1-\delta_{iS})$ and $p_{iS} = 1+T-\mu(1-\delta_{iS})$,

$$\alpha\left[1-\mu(1-\delta_{iS})-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}-nT\right]>0.$$

Hence, for $\alpha \leq \alpha_{iS}^{*}(\delta_{iS})$, the airline has the two choices, $p_{iS} = v$ and $p_{iS} = v+T$. Taking the difference between the profits under the two alternatives,

$$\begin{aligned}&T(\alpha+n)-(1-\alpha)\left[v-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]>0\\ &\leftrightarrow T>\hat{T}_{iS}(\delta_{iS})=\frac{(1-\alpha)\left[v-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]}{\alpha+n}.\end{aligned}$$

For $\alpha > \alpha_{iS}^{*}(\delta_{iS})$, the airline's choices are $p_{iS} = 1-\mu(1-\delta_{iS})$ and $p_{iS} = v+T$. By comparing the two profits,

$$\begin{aligned}&\alpha(1+n)\left\{1-\mu(1-\delta_{iS})\right\}-(\alpha+n)(v+T)-cn(1-\alpha)\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}<0\\ &\leftrightarrow T>\tilde{T}_{iS}(\delta_{iS})=\frac{\alpha(1+n)\left\{1-\mu(1-\delta_{iS})-v\right\}-n(1-\alpha)\left[v-c\left\{\delta_{iS}d_{iS}+(1-\delta_{iS})(d_{jS}+1)\right\}\right]}{\alpha+n}.\end{aligned}$$

Summarizing this discussion, we have Eqs. (16).

QED

As in Eq. (16.1), compared to the no subsidy case, for $\alpha \leq \alpha_{iS}^{*}(\delta_{iS})$, the airline excludes the air trip service consumption by type l travelers in two cities by choosing $p_{iS} = v+T$, 1 and 2 if the lump sum subsidy is sufficiently large. In contrast, for $\alpha >$

$\alpha_{iS}^*(\delta_{iS})$, type l residents in city S can consume the air trip service if the lump sum subsidy is large. This indicates that, with respect to the access to air trip service by l, the welfare effects of the lump sum subsidy depend on the population size of type b, α.

The equilibrium airfare, $\mathbf{p^L}(\boldsymbol{\delta}, T)$, is written as the function of the direct flight choice between 2 and S, δ_{2S}: that is, $\mathbf{p^L}(\boldsymbol{\delta}, T) = \mathbf{p^L}(\delta_{2S}, T)$. Substituting this into the profit, the airline's profit is also expressed as the function of δ_{2S}. Let us denote by $\delta_{2S}^L(T)$ the direct flight choice under the lump sum subsidy. Then, it is derived from:

$$\delta_{2S}^L(T) = \arg\max_{\delta_{2S}} \pi\left(\mathbf{p^L}(\delta_{2S}, T), \boldsymbol{\delta}, T\right).$$

Lemma 5 shows the airline's direct flight choice, $\delta_{2S}^L(T)$. Prior to summarizing, in order to simplify the notation, we define the thresholds as:

$$\bar{\alpha}_{2S}^* = \max\left\{\alpha_{2S}^*(0), \alpha_{2S}^*(1)\right\},\ \underline{\alpha}_{2S}^* = \min\left\{\alpha_{2S}^*(0), \alpha_{2S}^*(1)\right\},$$

$$\bar{T}_{2S}(\delta_{2S}) = \max\left\{\hat{T}_{2S}(\delta_{2S}), \tilde{T}_{2S}(\delta_{2S})\right\},\ \text{and}\ \underline{T}_{2S} = \min\left\{\tilde{T}_{2S}(0), \hat{T}_{2S}(1)\right\}.$$

Lemma 5

When the residents in city S receive the lump sum subsidy, the equilibrium direct flight choice, $\delta_{2S}^L(T)$, is summarized as:

$$\delta_{2S}^L(T) = \begin{cases} 0 \text{ if } F \geq F^L(T, \alpha, \mu), \\ 1 \text{ if } F < F^L(T, \alpha, \mu), \end{cases} \tag{16.1}$$

where

$$F^L(T,\alpha,\mu)=\begin{cases} c(1+n)(2-d) & \text{if } T \le \hat{T}_{2S}(0) \text{ and } \alpha \le \underline{\alpha}^*_{2S}, \\ (1+n)\{\alpha - v - c(\alpha d - 2)\} & \text{if } T \le \hat{T}_{2S}(0) \text{ and } \alpha^*_{2S}(1) < \alpha < \alpha^*_{2S}(0), \\ (1+n)\{v - \alpha(1-\mu) - c(d-2\alpha)\} & \text{if } T \le \underline{T}_{2S} \text{ and } \alpha^*_{2S}(0) < \alpha < \alpha^*_{2S}(1), \\ \alpha(1+n)\{\mu + c(2-d)\} & \text{if } T \le \min\{\hat{T}_{2S}(0), \hat{T}_{2S}(1)\} \text{ and } \alpha \ge \bar{\alpha}^*_{2S}, \\ c(\alpha+n)(2-d) & \text{if } T \ge \max\{\bar{T}_{2S}(1), \bar{T}_{2S}(0)\}, \\ v(1-\alpha) - (\alpha+n)(T-2c) - cd(1+n) & \text{if } \bar{T}_{2S}(0) < T < \hat{T}_{2S}(1), \\ \alpha(1+n)(1-cd) - (\alpha+n)(v+T-2c) & \text{if } \bar{T}_{2S}(0) < T < \tilde{T}_{2S}(1), \\ (\alpha+n)(v+T-cd) - \alpha(1+n)(1-\mu-2c) & \text{if } \bar{T}_{2S}(1) < T < \tilde{T}_{2S}(0). \end{cases}$$

(16.2)

Proof: Following the similar process as in Lemma 3, we have Eqs. (16). Substituting Eq. (15.1) into the airline's profit and comparing $\delta_{2S} = 1$ and $\delta_{2S} = 0$, we have the threshold as in Eq. (16.2). By using Eq. (16.2), we have the airline's network choice as reported in Eq. (16.1).

QED

The threshold, $F^L(T, \alpha, \mu)$, in Eq. (16.2) shows that the effect of the lump sum subsidy on the direct flight choice is quite limited, and the lump sum subsidy makes the airline sustain direct flights more easily if $\bar{T}_{2S}(1) < T < \tilde{T}_{2S}(0)$. This condition is met if $\bar{T}_{2S}(1) = \hat{T}_{2S}(1) < \tilde{T}_{2S}(0) = \bar{T}_{2S}(0)$, and this is realized if $\mu \le \mu^*$ and $\alpha^*_{2S}(1) > \alpha \ge \alpha^*_{2S}(0)$. In such situation, the degree of the exclusion is mitigated if the airline provides the direct flight service. In other words, the lump sum subsidy relaxes both the degree of the exclusion and the undersupply of the direct flight service only if $\bar{T}_{2S}(1) < T <$

$\tilde{T}_{2S}(0)$.

4.2. Ad Valorem Subsidy

In this subsection, we consider the case where residents in city S receive the subsidy, τp_{iS}. Under this situation, residents in city S and other two cities differ in the willingness to pay. Namely, since residents in S receive the subsidy, their willingness to pay is $1/(1-\tau)$ times as large as that of residents in other two cities. By assuming $v/(1-\tau) < 1-\mu$, the air trip demand is computed as:

$$Q_{iS}(\mathbf{p},\tau) = \begin{cases} 1+n \text{ if } 0 \le p_{iS} \le v, \\ 1+\alpha n \text{ if } v < p_{iS} \le \dfrac{v}{1-\tau}, \\ \alpha(1+n) \text{ if } \dfrac{v}{1-\tau} < p_{iS} \le 1-\mu(1-\delta_{iS}), \\ \alpha n \text{ if } 1-\mu(1-\delta_{iS}) < p_{iS} \le \dfrac{1-\mu(1-\delta_{iS})}{1-\tau}. \end{cases} \tag{17}$$

By using Eq. (17), the airline's profit is computed as:

$$\pi(\mathbf{p},\boldsymbol{\delta},\tau) = \sum_{i=1,2} Q_{iS}(\mathbf{p},\tau)\left[p_{iS} - c\left\{\delta_{iS} l_{iS} + (1-\delta_{iS})(l_{jS}+1)\right\}\right] - F(1+\delta_{2S}).$$

The equilibrium airfare under the ad valorem subsidy, $\mathbf{p}^{\mathbf{A}}(\boldsymbol{\delta},\tau)$, is derive from

$$\mathbf{p}^{\mathbf{A}}(\boldsymbol{\delta},\tau) = \arg\max_{\mathbf{p}} \pi(\mathbf{p},\boldsymbol{\delta},\tau),$$

and Lemma 6 summarizes the equilibrium airfare, $\mathbf{p}^{\mathbf{A}}(\boldsymbol{\delta},\tau)$:

Lemma 6

When the ad valorem subsidy to residents in city S is introduced, the equilibrium airfare, $\mathbf{p}^{\mathbf{A}}(\boldsymbol{\delta},\tau)$*, is derived as:*

$$p_{iS}^{A}\left(\delta_{iS},\tau\right)=\begin{cases} v \text{ if } \alpha\le\alpha_{iS}^{*}\left(\delta_{iS}\right) \text{ and } \tau\le\hat{\tau}_{iS}\left(\delta_{iS}\right), \\ \dfrac{v}{1-\tau} \text{ if } \max\left\{\hat{\tau}_{iS}\left(\delta_{iS}\right),\tilde{\tau}_{iS}\left(\delta_{iS}\right),\breve{\tau}_{iS}\left(\delta_{iS}\right)\right\}<\tau<\dfrac{1-\mu-v}{1-\mu}, \\ 1-\mu\left(1-\delta_{iS}\right) \text{ if } \alpha_{iS}^{*}\left(\delta_{iS}\right)<\alpha \text{ and } \tau\le\min\left\{\tilde{\tau}_{iS}\left(\delta_{iS}\right),\ddot{\tau}_{iS}\left(\delta_{iS}\right)\right\}, \\ \dfrac{1-\mu\left(1-\delta_{iS}\right)}{1-\tau} \text{ if } \alpha_{iS}^{*}\left(\delta_{iS}\right)<\alpha \text{ and } \ddot{\tau}_{iS}\left(\delta_{iS}\right)<\tau\le\breve{\tau}_{iS}\left(\delta_{iS}\right). \end{cases}$$

(18.1)

where

$$\hat{\tau}_{iS}\left(\delta_{iS}\right)=1-\frac{v\left(\alpha+n\right)}{v\left(1+n\right)-c\left(1-\alpha\right)\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}}, \quad (18.2)$$

$$\tilde{\tau}_{iS}\left(\delta_{iS}\right)=1-\frac{v\left(\alpha+n\right)}{\alpha\left(1+n\right)\left\{1-\mu\left(1-\delta_{iS}\right)\right\}+cn\left(1-\alpha\right)\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}},$$

(18.3)

$$\breve{\tau}_{iS}\left(\delta_{iS}\right)=1-\frac{v\left(\alpha+n\right)-\alpha n\left\{1-\mu\left(1-\delta_{iS}\right)\right\}}{c\left\{\alpha+n\left(1-\alpha\right)\right\}\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}}, \quad (18.4)$$

$$\ddot{\tau}_{iS}\left(\delta_{iS}\right)=1-\frac{n\left\{1-\mu\left(1-\delta_{iS}\right)\right\}}{\left(1+n\right)\left\{1-\mu\left(1-\delta_{iS}\right)\right\}-c\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}}.$$

(18.5)

Proof: Following the similar procedures as in Lemmas 2 and 4, we have Eqs. (18). Specifically, by comparing the profits under the four alternative situations as in Eq. (17), we have the thresholds summarized in Eqs. (18). By using the thresholds, we have the airfare choice as in Eq. (18.1).

QED

As in Eq. (18.1), similar to the lump sum subsidy, the welfare effects of the ad valorem subsidy on travelers depend on the population size of type b, α. That is, if the ad valorem subsidy is sufficiently large, for $\alpha \leq \alpha_{iS}^*(\delta_{iS})$, type l in the two cities cannot consume the air trip service while, for $\alpha > \alpha_{iS}^*(\delta_{iS})$, type l in city S can make air trips to other two cities. Different from the lump sum subsidy, for $\breve{\tau}_{iS}(\delta_{iS}) \geq \tau > \ddot{\tau}_{iS}(\delta_{iS})$, other than type b in city S cannot utilize the air trip service to (from) S. It is, however, hard to imagine that this situation is realized; therefore, by comparing $\breve{\tau}_{iS}(\delta_{iS})$ and $\tilde{\tau}_{iS}(\delta_{iS})$, we have Lemma 7.

Lemma 7

If the reservation price of type l is sufficiently large ($1 > v > \breve{v}_{iS}(\delta_{iS})$), $\tilde{\tau}_{iS}(\delta_{iS}) > \breve{\tau}_{iS}(\delta_{iS})$ where $\breve{v}_{iS}(\delta_{iS})$ is defined as:

$$\breve{v}_{iS}(\delta_{iS}) = \frac{n\{1-\mu(1-\delta_{iS})\}\left[\alpha(1+n)\{1-\mu(1-\delta_{iS})\} - cn(1-\alpha)\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]}{(\alpha+n)\left[1-\mu(1-\delta_{iS}) - c\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]}.$$

Proof: By comparing Eqs. (18.3) and (18.4),

$$\begin{aligned}&\tilde{\tau}_{iS}(\delta_{iS}) > \breve{\tau}_{iS}(\delta_{iS}) \\ &\leftrightarrow v > \breve{v}_{iS}(\delta_{iS}) = \frac{n\{1-\mu(1-\delta_{iS})\}\left[\alpha(1+n)\{1-\mu(1-\delta_{iS})\} - cn(1-\alpha)\left\{\delta_{iS}l_{iS} + (1-\delta_{iS})(l_{jS}+1)\right\}\right]}{(\alpha+n)\left[1-\mu(1-\delta_{iS}) - c\left\{\delta_{iS}l_{iS} + (1-\delta_{iS})(l_{jS}+1)\right\}\right]}.\end{aligned}$$

QED

Hereafter, we consider the case where $v > \breve{v}_{iS}(\delta_{iS})$. Under this assumption, Eq. (18.1) is rewritten as:

$$p_{iS}^{A}\left(\delta_{iS},\tau\right)=\begin{cases} v \text{ if } \alpha \le \alpha_{iS}^{*}\left(\delta_{iS}\right) \text{ and } \tau \le \hat{\tau}_{iS}\left(\delta_{iS}\right), \\ \dfrac{v}{1-\tau} \text{ if } \max\left\{\hat{\tau}_{iS}\left(\delta_{iS}\right),\tilde{\tau}_{iS}\left(\delta_{iS}\right)\right\} < \tau < \dfrac{1-\mu-v}{1-\mu}, \\ 1-\mu\left(1-\delta_{iS}\right) \text{ if } \alpha_{iS}^{*}\left(\delta_{iS}\right) < \alpha \text{ and } \tau \le \tilde{\tau}_{iS}\left(\delta_{iS}\right). \end{cases}$$

(19)

Substituting Eq. (19) into the airline's profit, we can rewrite the profit as the function of δ_{2S}. Let us denote by $\delta_{2S}^{A}(\tau)$ the airline's direct flight choice, and it is obtained as the solution of the following problem:

$$\delta_{2S}^{A}\left(\tau\right)=\arg\max_{\delta_{2S}}\pi\left(\mathbf{p}^{\mathbf{A}}\left(\delta_{2S},\tau\right),\boldsymbol{\delta},\tau\right).$$

Prior to summarizing the direct flight choice, $\delta_{2S}^{A}(\tau)$, we define the two thresholds as follows:

$$\overline{\tau}_{2S}\left(\delta_{2S}\right)=\max\left\{\hat{\tau}_{2S}\left(\delta_{2S}\right),\tilde{\tau}_{2S}\left(\delta_{2S}\right)\right\} \text{ and } \underline{\tau}_{2S}=\min\left\{\tilde{\tau}_{2S}\left(0\right),\hat{\tau}_{2S}\left(1\right)\right\}.$$

By using these thresholds, Lemma 8 shows $\delta_{2S}^{A}(\tau)$:

Lemma 8

When the residents in city S receive the ad valorem subsidy, the equilibrium direct flight choice, $\delta_{2S}^{A}(\tau)$, is summarized as:

$$\delta_{2S}^{A}\left(\tau\right)=\begin{cases} 0 \text{ if } F \ge F^{A}\left(\tau,\alpha,\mu\right), \\ 1 \text{ if } F < F^{A}\left(\tau,\alpha,\mu\right), \end{cases} \tag{20.1}$$

where

$$F^A(\tau,\alpha,\mu)=\begin{cases} c(1+n)(2-d) \text{ if } \tau \le \hat{\tau}_{2S}(0) \text{ and } \alpha \le \underline{\alpha}^*_{2S}, \\ (1+n)\{\alpha - v - c(\alpha d - 2)\} \text{ if } \tau \le \hat{\tau}_{2S}(0) \text{ and } \alpha^*_{2S}(1) < \alpha < \alpha^*_{2S}(0), \\ (1+n)\{v - \alpha(1-\mu) - c(d-2\alpha)\} \text{ if } \tau \le \underline{\tau}_{2S} \text{ and } \alpha^*_{2S}(0) < \alpha < \alpha^*_{2S}(1), \\ \alpha(1+n)\{\mu + c(2-d)\} \text{ if } \tau \le \min\{\tilde{\tau}_{2S}(0), \tilde{\tau}_{2S}(1)\} \text{ and } \alpha \ge \overline{\alpha}^*_{2S}, \\ c(\alpha+n)(2-d) \text{ if } \tau \ge \max\{\overline{\tau}_{2S}(0), \overline{\tau}_{2S}(1)\}, \\ (1+n)(v-cd) - (\alpha+n)\left(\dfrac{v}{1-\tau} - 2c\right) \text{ if } \overline{\tau}_{2S}(0) < \tau < \hat{\tau}_{2S}(1), \\ \alpha(1+n)(1-cd) - (\alpha+n)\left(\dfrac{v}{1-\tau} - 2c\right) \text{ if } \overline{\tau}_{2S}(0) < \tau < \overline{\tau}_{2S}(1), \\ (\alpha+n)\left(\dfrac{v}{1-\tau} - cd\right) - \alpha(1+n)(1-\mu-2c) \text{ if } \overline{\tau}_{2S}(1) < \tau < \overline{\tau}_{2S}(0). \end{cases}$$

(20.2)

Proof: Following the similar steps as in Lemma 3, according to the comparison of profits between $\delta_{2S} = 1$ and $\delta_{2S} = 0$, we have the thresholds in Eq. (20.2). By using the thresholds in (20.2), we obtain the equilibrium direct flight choice in (20.1).

QED

As in Eq. (20.2), the same as the lump sum subsidy, the effect of the ad valorem subsidy on the direct flight choice is quite limited. For $\alpha^*_{2S}(0) < \alpha$ and $\overline{\tau}_{2S}(1) < \tau \le \overline{\tau}_{2S}(0)$, however, the ad valorem subsidy has a positive impact on the sustainability of the direct flight service between 2 and S. Under this circumstance, once the direct flight service is sustained, the degree of the exclusion is mitigated since $p^A_{2S}(1,\tau) = v/(1-\tau) <$

$p_{2S}^{A}(0,\tau) = 1 - \mu$. In other words, for this situation, the ad valorem subsidy enhances both the direct flight service and the air trip demand.

5. Lump Sum Subsidy vs. Ad Valorem Subsidy

This section evaluates the effects of the two alternative subsidies on the airline's choices on the airfare and the direct flight service. As in Section 3, the market power of the airline causes the undersupply of the air trip service. In addition, the undersupply makes the airline cease the direct flight service more easily than the efficient level since the airline underestimates the benefit of sustaining the direct flight service. In this section, we address the question how the two alternative subsidies, lump sum and ad valorem, mitigate these two inefficiencies. In order to compare these two policies, we set the following assumption:

$$\tau v = T\left(\delta_{iS}\right). \tag{21}$$

First, we rearrange the airline's choice under the lump sum subsidy as the function of τ. Specifically, Lemma 4 is rewritten as Lemma 9:

Lemma 9

When the lump sum subsidy given by Eq. (21) *is introduced, the airline's airfare* $\mathbf{p^L}(\boldsymbol{\delta},\tau)$

is derived as:

$$p_{iS}^{L}\left(\delta_{iS},\tau\right)=\begin{cases} v \text{ if } \alpha \le \alpha_{iS}^{*}\left(\delta_{iS}\right) \text{ and } \tau \le \hat{\tau}_{iS}^{L}\left(\delta_{iS}\right), \\ v+\tau\left\{\alpha+v\left(1-\alpha\right)\right\} \text{ if } \overline{\tau}_{iS}^{L}\left(\delta_{iS}\right)<\tau<\dfrac{1-\mu}{v}-1, \\ 1-\mu\left(1-\delta_{iS}\right) \text{ if } \alpha_{iS}^{*}\left(\delta_{iS}\right)<\alpha \text{ and } \tau \le \tilde{\tau}_{iS}^{L}\left(\delta_{iS}\right). \end{cases}$$

(22.1)

where

$$\hat{\tau}_{iS}^{L}\left(\delta_{iS}\right)=\frac{\hat{T}_{iS}\left(\delta_{iS}\right)}{v}=\frac{\left(1-\alpha\right)\left[v-c\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}\right]}{v\left(\alpha+n\right)},$$

(22.2)

$$\begin{aligned}\tilde{\tau}_{iS}^{L}\left(\delta_{iS}\right)&=\frac{\tilde{T}_{iS}\left(\delta_{iS}\right)}{v}\\ &=\frac{\alpha\left(1+n\right)\left\{1-\mu\left(1-\delta_{iS}\right)-v\right\}-n\left(1-\alpha\right)\left[v-c\left\{\delta_{iS}d_{iS}+\left(1-\delta_{iS}\right)\left(d_{jS}+1\right)\right\}\right]}{v\left(\alpha+n\right)}.\end{aligned}$$

(22.3)

Proof: As in Lemma 4, for $\alpha \le \alpha_{iS}^{*}(\delta_{iS})$, the airline has the two alternatives such as $p_{iS}=v$ and $p_{iS}=v+T(\delta_{iS})$. Taking the difference in the profits between the two alternatives and solving this for τ, we obtain the result such that the airline prefers $p_{iS}=v+T$ if $\tau>\hat{\tau}_{iS}^{L}(\delta_{iS})$ as in Eq. (22.2). For $\alpha>\alpha_{iS}^{*}(\delta_{iS})$, the airline has the two choices such as $p_{iS}=1-\mu(1-\delta_{iS})$ and $p_{iS}=v+T(\delta_{iS})$. By comparing the profits, we can conclude that, if $\tau>\tilde{\tau}_{iS}^{L}(\delta_{iS})$, the airline strictly prefers $p_{iS}=v+T(\delta_{iS})$.

QED

In addition, the direct flight choice in Lemma 5 is rewritten as Lemma 10:

Lemma 10

When the residents of city S *receive the lump sum subsidy characterized by Eq.* (21)*, the airline's direct flight choice,* $\delta_{2S}^{L}(\tau)$*, is characterized as:*

$$\delta_{2S}^{L}(\tau)=\begin{cases}0 \text{ if } F \geq F^{L}(\tau,\alpha,\mu),\\ 1 \text{ if } F < F^{L}(\tau,\alpha,\mu),\end{cases} \tag{23.1}$$

where

$$F^{L}(\tau,\alpha,\mu)=\begin{cases} c(1+n)(2-d) \text{ if } \tau \leq \hat{\tau}_{2S}^{L}(0) \text{ and } \alpha \leq \underline{\alpha}_{2S}^{*},\\ (1+n)\{\alpha-v-c(\alpha d-2)\} \text{ if } \tau \leq \hat{\tau}_{2S}^{L}(0) \text{ and } \alpha_{2S}^{*}(1)<\alpha<\alpha_{2S}^{*}(0),\\ (1+n)\{v-\alpha(1-\mu)-c(d-2\alpha)\} \text{ if } \tau \leq \underline{\tau}_{2S}^{L} \text{ and } \alpha_{2S}^{*}(0)<\alpha<\alpha_{2S}^{*}(1),\\ \alpha(1+n)\{\mu+c(2-d)\} \text{ if } \tau \leq \min\{\hat{\tau}_{2S}^{L}(0),\hat{\tau}_{2S}^{L}(1)\} \text{ and } \alpha \geq \bar{\alpha}_{2S}^{*},\\ c(\alpha+n)(2-d) \text{ if } \tau \geq \max\{\bar{\tau}_{2S}^{L}(1),\bar{\tau}_{2S}^{L}(0)\},\\ v(1-\alpha)-(\alpha+n)(\tau v-2c)-cd(1+n) \text{ if } \bar{\tau}_{2S}^{L}(0)<\tau<\hat{\tau}_{2S}^{L}(1),\\ \alpha(1+n)(1-cl)-(\alpha+n)\{v(1+\tau)-2c\} \text{ if } \bar{\tau}_{2S}^{L}(0)<\tau<\tilde{\tau}_{2S}^{L}(1),\\ (\alpha+n)(\{v(1+\tau)-cd\})-\alpha(1+n)(1-\mu-2c) \text{ if } \bar{\tau}_{2S}^{L}(1)<\tau<\tilde{\tau}_{2S}^{L}(0),\end{cases}$$

(23.2)

$$\bar{\tau}_{2S}^{L}(\delta_{2S})=\max\{\hat{\tau}_{2S}^{L}(\delta_{2S}),\tilde{\tau}_{2S}^{L}(\delta_{2S})\}, \text{ and } \underline{\tau}_{2S}^{L}=\min\{\tilde{\tau}_{2S}^{L}(0),\hat{\tau}_{2S}^{L}(1)\}.$$

(23.3)

Proof: By using Eq. (21), we rewrite Eq. (17.2) with respect to τ, and obtain Eq. (23.2).

QED

By using these results, we obtain Proposition 2, which compares the airline's decision on the airfare under the alternative subsidies.

Proposition 2

The lump sum subsidy mitigates the degree of the exclusion ($Q_{iS}(\mathbf{p^A}(\tau,\boldsymbol{\delta}),\tau) \leq Q_{iS}(\mathbf{p^L}(\tau,\boldsymbol{\delta}),\tau)$) *for* $0 < \alpha \leq \alpha_{iS}^*(\delta_{iS})$ *while it reinforces the degree of the exclusion* ($Q_{iS}(\mathbf{p^A}(\tau,\boldsymbol{\delta}),\tau) \geq Q_{iS}(\mathbf{p^L}(\tau,\boldsymbol{\delta}),\tau)$) *for* $\alpha_{iS}^*(\delta_{iS}) < \alpha < 1$.

Proof: For $0 < \alpha \leq \alpha_{iS}^*(\delta_{iS})$, by comparing two thresholds, $\hat{\tau}_{iS}(\delta_{iS})$ and $\hat{\tau}_{iS}^L(\delta_{iS})$,

$$\hat{\tau}_{iS}(\delta_{iS}) - \hat{\tau}_{iS}^L(\delta_{iS}) = -\frac{(1-\alpha)^2\left[v - c\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]^2}{v(\alpha+n)\left[v(1+n) - c(1-\alpha)\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]} < 0.$$

This implies that, for $0 < \alpha \leq \alpha_{iS}^*(\delta_{iS})$, $\hat{\tau}_{iS}(\delta_{iS}) < \hat{\tau}_{iS}^L(\delta_{iS})$. For $\hat{\tau}_{iS}(\delta_{iS}) < \tau < \hat{\tau}_{iS}^L(\delta_{iS})$, $p_{iS}^A = v/(1-\tau) > p_{iS}^L = v$. For $\alpha_{iS}^*(\delta_{iS}) < \alpha < 1$, according to the comparison of two thresholds, $\tilde{\tau}_{iS}(\delta_{iS})$ and $\tilde{\tau}_{iS}^L(\delta_{iS})$, we have:

$$\begin{aligned}&\tilde{\tau}_{iS}(\delta_{iS}) - \tilde{\tau}_{iS}^L(\delta_{iS}) \\ &\quad = -\frac{\left[v(\alpha+n) + \alpha(1+n)\{1-\mu(1-\delta_{iS})\} - c\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]^2}{v(\alpha+n)\left[\alpha(1+n)\{1-\mu(1-\delta_{iS})\} - cn(1-\alpha)\left\{\delta_{iS}d_{iS} + (1-\delta_{iS})(d_{jS}+1)\right\}\right]} < 0.\end{aligned}$$

This indicates that, for $\alpha_{iS}^*(\delta_{iS}) < \alpha < 1$, $\tilde{\tau}_{iS}(\delta_{iS}) < \tilde{\tau}_{iS}^L(\delta_{iS})$. Hence, for $\tilde{\tau}_{iS}(\delta_{iS}) < \tau < \tilde{\tau}_{iS}^L(\delta_{iS})$, $p_{iS}^A = v/(1-\tau) < p_{iS}^L = 1 - \mu(1-\delta_{iS})$.

QED

Given τ, under the ad valorem subsidy, the airline can raise the airfare more easily than under the lump sum subsidy: that is,

$$p_{iS}^{L} = v + T = v(1+\tau) < \frac{v}{1-\tau} = p_{iS}^{A}. \tag{24}$$

For $0 < \alpha \leq \alpha_{iS}^{*}(\delta_{iS})$, since the population of type b is sufficiently small, the airline is less motivated to raise the airfare. Furthermore, the lump sum subsidy makes the gain of raising airfare smaller than the ad valorem does. Consequently, for $0 < \alpha \leq \alpha_{iS}^{*}(\delta_{iS})$, the lump sum subsidy mitigates the degree of the exclusion. For $\alpha_{iS}^{*}(\delta_{iS}) < \alpha < 1$, in contrast, the ad valorem subsidy allows the more population to consume the air trip service.

Finally, we state Proposition 3, which compares the effects on the direct flight choice of the two alternative subsidy schemes.

Proposition 3

Let us define by $\tilde{\mu}$ as:

$$\tilde{\mu} = \frac{cn(1-\alpha)(2-d)}{\alpha(1+n)}.$$

The lump sum subsidy enhances the direct flight service between airports 2 and S ($F^{A}(\tau,\alpha,\mu) < F^{L}(\tau,\alpha,\mu)$) if $\alpha < \underline{\alpha}_{2S}^{*}$, or $\alpha > \overline{\alpha}_{2S}^{*}$ and $\mu \geq \tilde{\mu}$: otherwise, with respect to the direct flight service, the ad valorem subsidy is the second-best ($F^{A}(\tau,\alpha,\mu) > F^{L}(\tau,\alpha,\mu)$).

Proof: For $\alpha < \underline{\alpha}_{2S}^{*}$, as in Proposition 2, $\hat{\tau}_{iS}(\delta_{iS}) < \hat{\tau}_{iS}^{L}(\delta_{iS})$. For $\tau \leq \hat{\tau}_{iS}(0)$ and $\hat{\tau}_{iS}^{L}(1) \leq \tau$, the airline's direct flight choice is identical between the two subsidies. For $\hat{\tau}_{iS}(0) < \tau < \hat{\tau}_{iS}^{L}(1)$, however, the airline's thresholds differ, and we take the differences in the thresholds as follows:

$$
F^{A}(\tau,\alpha,\mu) - F^{L}(\tau,\alpha,\mu) \\
= \begin{cases} v\left(1+n-\dfrac{\alpha+n}{1-\tau}\right) - 2c(1-\alpha) & \text{for } \hat{\tau}_{2S}(0) < \tau < \min\left\{\hat{\tau}_{2S}(1), \hat{\tau}_{2S}^{L}(0)\right\}, \\ -\dfrac{v\tau^{2}(\alpha+n)}{1-\tau} & \text{for } \hat{\tau}_{2S}(1) < \tau < \hat{\tau}_{2S}^{L}(0), \\ -c(1-\alpha)(2-d) & \text{for } \hat{\tau}_{2S}^{L}(0) < \tau < \hat{\tau}_{2S}(1), \\ cd(1-\alpha) - v\{1-\alpha-\tau(\alpha+n)\} & \text{for } \max\left\{\hat{\tau}_{2S}(1), \hat{\tau}_{2S}^{L}(0)\right\} < \tau < \hat{\tau}_{2S}^{L}(1). \end{cases}
$$

Since we focus on the case where $\tilde{\tau}_{iS}(0) < \tau < \tilde{\tau}_{iS}^{L}(1)$, it is easy to confirm $F^{A}(\tau,\alpha,\mu) < F^{L}(\tau,\alpha,\mu)$. For $\alpha > \overline{\alpha}_{2S}^{*}$, $\tilde{\tau}_{iS}(\delta_{iS}) < \tilde{\tau}_{iS}^{L}(\delta_{iS})$ as shown in Proposition 2. By comparing the thresholds on the subsidy rate, we have:

$$
\tilde{\tau}_{2S}(0) > \tilde{\tau}_{2S}(1) \text{ and } \tilde{\tau}_{2S}^{L}(0) > \tilde{\tau}_{2S}^{L}(1) \leftrightarrow \mu < \tilde{\mu} = \frac{cn(1-\alpha)(2-d)}{\alpha(1+n)}.
$$

In case of $\mu < \tilde{\mu}$, according to the comparison of $F^{A}(\tau,\alpha,\mu)$ and $F^{L}(\tau,\alpha,\mu)$, we have:

$$F^A(\tau,\alpha,\mu)-F^L(\tau,\alpha,\mu)$$

$$=\begin{cases} v\left(\dfrac{\alpha+n}{1-\tau}-1+n\right)-cd(1-\alpha) \text{ for } \tilde{\tau}_{2S}(1)<\tau<\min\left\{\tilde{\tau}_{2S}(0),\tilde{\tau}_{2S}^L(1)\right\}, \\ cn(1-\alpha)(2-d)-\alpha\mu(1+n) \text{ for } \tilde{\tau}_{2S}(0)<\tau<\tilde{\tau}_{2S}^L(1), \\ \dfrac{v\tau^2(\alpha+n)}{1-\tau} \text{ for } \tilde{\tau}_{2S}^L(1)<\tau<\tilde{\tau}_{2S}(0), \\ \alpha(1+n)(1-\mu)+2cn(1-\alpha)-v(\alpha+n)(1+\tau) \text{ for } \max\left\{\tilde{\tau}_{2S}(0),\tilde{\tau}_{2S}^L(1)\right\}<\tau<\tilde{\tau}_{2S}^L(0). \end{cases}$$

Since $\mu<\tilde{\mu}$ and $\tilde{\tau}_{iS}(1)<\tau<\tilde{\tau}_{iS}^L(0)$, $F^A(\tau,\alpha,\mu)>F^L(\tau,\alpha,\mu)$. For $\mu\geq\tilde{\mu}$, $\tilde{\tau}_{iS}(0)<\tau<\tilde{\tau}_{iS}^L(1)$. In this case, by taking the difference of the thresholds, we have:

$$F^A(\tau,\alpha,\mu)-F^L(\tau,\alpha,\mu)$$

$$=\begin{cases} \alpha(1+n)(1-\mu)+2cn(1-\alpha)-\dfrac{v(\alpha+n)}{1-\tau} \text{ for } \tilde{\tau}_{2S}(0)<\tau<\min\left\{\tilde{\tau}_{2S}(1),\tilde{\tau}_{2S}^L(0)\right\}, \\ cn(1-\alpha)(2-d)-\alpha\mu(1+n) \text{ for } \tilde{\tau}_{2S}(1)<\tau<\tilde{\tau}_{2S}^L(0), \\ -\dfrac{v\tau^2(\alpha+n)}{1-\tau} \text{ for } \tilde{\tau}_{2S}^L(0)<\tau<\tilde{\tau}_{2S}(1), \\ v(\alpha+n)(1+\tau)-\alpha(1+n)-cdn(1-\alpha) \text{ for } \max\left\{\tilde{\tau}_{2S}(0),\tilde{\tau}_{2S}^L(1)\right\}<\tau<\tilde{\tau}_{2S}^L(0). \end{cases}$$

Since $\mu\geq\tilde{\mu}$ and $\tilde{\tau}_{iS}(0)<\tau<\tilde{\tau}_{iS}^L(1)$, $F^A(\tau,\alpha,\mu)<F^L(\tau,\alpha,\mu)$.

QED

As in Proposition 3, in general, the lump sum subsidy may become the second-best to correct the undersupply of the direct flight service between 2 and S. We take a closer look at the mechanism behind this result. First, for $\alpha<\underline{\alpha}_{2S}^*$, independent from the subsidy type, the airline can easily raise the airfare in case of no direct flights between 2 and S ($\delta_{2S}=0$). Furthermore, as in Eq. (24), the ad valorem assures the larger benefit from raising the airfare than the lump sum. As a result, under the ad valorem, compared to the optimum, the airline overestimates the benefit of raising the airfare and underestimates the gain of sustaining the direct flight service between 2 and S. To put it differently, for $\alpha < \underline{\alpha}_{2S}^{*}$,

For $\alpha > \overline{\alpha}_{2S}^{*}$, if $\mu \geq \tilde{\mu}$, due to the high transit cost, it is much easier to cut the airfare when the airline does not provide the direct flight service. Furthermore, as in Eq. (24), under the ad valorem subsidy, the airline can easily mitigate the loss due to the reduction in the airfare. In other words, with respect to the undersupply of the air trip service, the ad valorem subsidy is the second-best. In contrast, however, due to the high proportion of type b and the high transit cost, the cost of choosing no direct flight is increased; therefore, with respect to the undersupply of the direct flight service, the lump sum is the second-best. If $\mu < \tilde{\mu}$, since the transit cost is negligible, the ad valorem subsidy corrects the airline's incentives to exclude the air trip consumption by l in cities other than S and to cease the direct flight service between 2 and S.

5. Conclusion

This paper evaluates the effect of the two alternative subsidies on the airline's choices of the airfare and the direct flight service. Through the analysis, we obtain the following results. First, when no subsidies are introduced, since the airline abuses its market power, the undersupply of the air trip service emerges. Furthermore, due to the undersupply of the air trip service, the airline ceases the direct flight service for thin demand routes more easily than the efficient level since it underestimates the benefit of providing the direct flight service. Once the subsidies are introduced, the undersupply of the air trip service is may mitigated if the subsidy is sufficiently large.

With respect to the direct flight service, however, since it is difficult to compare the effect of the subsidy on the airline's choice, we compare the effects when the public expenditure of the subsidy is equivalent between the two regimes. By comparing the two subsidies, when the proportion of the travelers with the high willingness to pay is sufficiently high and the transit cost is negligible, the ad valorem subsidy corrects both undersupplies of the air trip and of the direct flight more easily than the lump sum. In contrast, if the proportion of the travelers with the high willingness to pay is sufficiently low, the lump sum subsidy remedies the two inefficiencies. These results generated from the fact such that the ad valorem subsidy assures the larger airfare revenue to the airline.

In contrast to these, when the proportion of the travelers with the high willingness to pay is sufficiently high and the transit cost is sufficiently large, the ad valorem subsidy corrects the undersupply of the air trip while the lump sum mitigates the undersupply of the direct flight service. The former comes from the same mechanism as in other cases described above. In contrast, the latter comes from the fact such that, due to the parameters, the airline must incur the more cost of choosing no direct flight.

In this model, since the decentralized decision-making by the airline generates the two inefficiencies with respect to the direct flight and the air trip services, the subsidy has a justification to improve the efficiency. Since, as discussed above, the proportion of the high willingness to pay is the key factor to determine the second-best subsidy scheme, it is necessary to clear the proportion for each thin demand route. Furthermore, since the network externality is absent in this model, our model may underestimate the airline's benefit of sustaining the direct flight; therefore, it is also necessary to extend our model by introducing such externality.

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