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The Political Merger under Alternative Rules

TERAJI, Yusuke
Faculty of Economics, Tezukayama University

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Tezukayama University
Research Institute for Economics and Business
7-1-1 Tezukayama, Nara 631-8501, Japan

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Abstract:

Several countries intervene in the political merger of local governments by using several kinds of policy tools. Among the several policy tools, we focus on the revision of rules, and address the question what type of rule replicates the efficient configuration of the economy. In order to deal with this problem, we develop a model that incorporates the tradeoff of the merger for residents: namely, the scale economy, the increase in the heterogeneity among residents, and the rise in the access cost to the public facility. By using this model, we compare alternative rules such as: Majority Voting; Municipal Request; and Central Approval. Through the comparison of these three rules, it is shown that the Majority Voting makes the merger relatively hard to realize whereas the Municipal Request makes it relatively easy to realize. The efficient rule, however, varies with the situation: namely if both the access cost and the degree of the heterogeneity are relatively small, the Municipal Request becomes the efficient rule.

Keywords: Political Integration, Coalition Formation, Intervention, Revision of Rules

1. Introduction

During the last two decades, several countries have experienced the reorganization of local governments through the merger. In most cases, the upper-tier government (central or province) intervene in the reorganization by implementing several policy tools such as the conditional subsidy[1] and the revision of rules. In the Japanese case, the central government has mainly used the conditional subsidy to enhance the merger of local

[1] For example, the Japanese Central Government excludes aid of 400 million yen to each merged municipality for the sake of correcting the gaps within its new boundary. In addition, the government also subsidizes the localities to ensure fiscal requirements due to the merger, with a ceiling of 300 million yen. Moreover, exceptions are allowed for aggregate subsidies if the merger is approved.

governments. In addition, Japanese central government has revised the rule by giving an initiative of merger to residents. This type of revision is at the one extreme, and at the other extreme, some upper-tier governments has chosen the fiat (for example, Montreal and Toronto). Although these revisions are aimed at enhancing the merger by lowering the costs of merger for individual local governments, sometimes it has resulted in failure. In the Japanese case, even though the number of merger is more than 500 from 1999 to 2005, none of merger is originated from the residents' initiative. Furthermore, in case of Montreal, the merger in 2002 is realized through the fiat by Quebec Province, but two years later, because of the severe protest by residents, the mega Montreal is demerged into 15 municipalities. These two examples reflect that the revision of rules did not always lower the cost of the merger. In general, the merger may improve the efficiency in the public good provision, but it also generates the cost such as the increase in the heterogeneity among residents and the rise in the access or the delivery cost to the public facilities. Therefore, by incorporating this tradeoff, it is important to deal with the problem how the revision of rules affects the behavior of residents and local governments.

After the seminal work of Tiebout (1956), much literature investigates questions such as: whether the choice of local governments on their sizes deviates from the optimum;

and whether it is necessary for the upper-tier governments to intervene in the choice of local governments. Based on Tiebout (1956), it is shown that the optimal size is achieved through decentralized decision-making by local governments (e.g. Berglas 1976; Stiglitz 1977; Arnott 1979; Arnott and Stiglitz 1979; Scotchmer and Wooders 1987; Cornes and Sandler 1996).[2] Although, at a glance, the literature suggests that laissez-faire reorganization leads to optimal allocation, their results hinge on the assumption: free mobility of club members (or residents). With immobile residents, Cremer et al. (1985) work on optimal locations of facilities and whether those locations are realized under decentralized setting. They establish a result such that decentralized decision-making divides an economy into jurisdictions inefficiently small size. Alesina and Spolaore (1997) obtain the same results as Cremer et al. (1985) and show that residents enjoying the lowest welfare at the optimum have an incentive to deviate from the optimum.

In the context of the merger, these results in Cremer et al. (1985) and Alesina and Spolaore (1997) indicate that the configuration achieved through the decentralized negotiation deviates from the optimal. Their model, however, allows that the boundary of a jurisdiction is redrawn flexible and instantaneously, and each local government

[2] These studies have shown that when the residents are mobile, optimal sized clubs or local jurisdictions are replicated through a system of profit-maximizing clubs or developers.

cannot choose whether to redraw its boundaries. In contrast, in case of the merger, the boundary of a newly formed jurisdiction is drawn based on the boundaries of jurisdictions joining to, and each local government can determine whether to merge. In this contest, similar situation is studied in literature on political integration (Bolton and Roland 1997; Ellingsen 1998).[3] They develop a two-country model, and show that the decentralized decision making on the integration cannot replicate the optimum. In these models, each government chooses only whether to integrate with its neighboring country while, in case of the reorganization of local governments, each government chooses its counterpart as well as whether to merge. In this regard, Desmet et al. (2011) extend the model by allowing that each local government can choose its counterpart for the merger.

It is widely shown that, under the decentralized negotiation, it is relatively difficult to reach a conclusion of the merger compared to under the benevolent social planner. Few studies, however, deal with the problem how the upper-tier government intervenes in the negotiation of local governments. Based on the model of Desmet et al. (2011), Wesse (2011) studies the welfare effects of the conditional subsidy, and numerically shows that

[3] Bolton and Roland (1997) develop a model of two countries by incorporating the disparity in income distribution across countries, and show that separation is preferred even though the unification is always efficient. Ellingsen (1998) studies whether or not the decision making by countries achieves the same result as an efficient case, and he establishes the result such that integration under decentralized decision making is less realized than the efficient case.

subsidizing relatively rich jurisdiction improves the efficiency through enhancing the merger. In case of the revision of rules, Jehiel and Scotchmer (1997) investigate the welfare effects of the revision of rules, but their focus is on the unilateral annexation rather than the political merger of jurisdictions. This paper deals with the revision of rules, and addresses the problem what type of rules replicates the efficient level of the merger.

When concerning about the rules for the reorganization of local governments, there is a wide variety of rules according to complexity of procedures for reorganization. In case of United States, Lindsey (2004) reports that each of 50 states sets different types of rules as in Table 1. Under the last type of rules in Table 1, “Municipal Request”, the merger is accomplished once a single jurisdiction requests it while, under the first three types of rules, the reorganization is realized if the state government accepts it. Based on this variety of rules implemented in the United States, we set up three alternative rules: such as i) Majority Voting; ii) Municipal Request; and iii) Central Approval. Under Rule i), the merger of local governments takes place if all connected jurisdictions join up to prove the merger by referendum while in case of Rule ii), the merger takes place if a single jurisdiction requests it. Under Rule iii), the central government sets the referendum on the change of the political configuration, and the configuration will change once the majority of the economy supports it. According to the comparison of these three alternative rules, this paper aims at evaluating the welfare effects of rule setting for political merger. That is, we address the question of which rule achieves more efficient outcome of the merger. In order to deal with this problem, we develop a model that incorporates the tradeoff between the scale economy and the costs of the merger such as the increase in the heterogeneity among residents and the rise in the access cost to the public facility.

<<Table 1: About HERE>>

This paper is organized as follows. In Section 2, we describe the model, and in Section 3, we investigate the political merger under three alternative rules: i) Majority Voting; ii) Municipal Request; iii) Central Approval. In order to evaluate the efficiency of alternative three rules, Section 4 first studies the benchmark case in which the configuration is determined in order to maximize the economic welfare. Then, we address the evaluation of three alternative rules by comparing three types of configurations derived in Section 3 with the benchmark. Finally, Section 6 states concluding remarks.

2. The Model

2.1 The Basic Setting

Let us represent the economy by a linear space with the length, L, and the population of this economy is equal to N. Locations in this economy are expressed as distances from the left end of the line. This economy is equally divided into three partitions; therefore the land in partition i (i=1, 2, 3), L_i, is equal to $\bar{L} = L/3$, and is represented by the interval $x \in [L_{j-1}^b, L_j^b]$ where $L_j^b = j \times \bar{L}$. All the land in this economy is owned publicly: that is, the aggregate land rent is equally shared among all households. In addition, we assume that the land is only used for the residence, and that each household consumes one unit of land. According to these assumptions, we have $N = L$, and in each partition j, the population is equal to $\bar{N} = N/3$. Furthermore, the locations in partition j are given by $x \in [(j-1)\bar{N}, j\bar{N}]$.

<<Figure 1: About HERE>>

The households in the economy are heterogeneous in preferences toward the public good. Each household's preference intensity is represented by θ, and we call them type θ. We assume that as θ increases, the household's preference toward the public good becomes stronger. The preference parameter θ is uniformly distributed along the segment $\theta \in [\underline{\theta}, \bar{\theta}]$ with the density ρ. For the sake of the simplicity, we set the distribution of θ as $\theta \in [\theta_{j-1}^b, \theta_j^b]$ where $\theta_j^b = (3-j)\underline{\theta}/3 + j\bar{\theta}/3$.[4]

[4] Although this assumption is quite restrictive, in this model, the differences among locations are

A town is formed in each partition, and hereafter we call Town *i* whose jurisdiction is equal to partition *i*. Furthermore, we denote by Town *jk* a local government that governs two partitions *j* and *k*. Each Town *i* has a government, and governments make three kinds of choices according to the referendum such as, the location of the facility, x_i; the amount of the public good, g_i; the head tax for the public good provision, τ_i; and whether to merge. Table 2 summarizes the possible merger patterns and political configurations. In Table 2, σ_y ($y = 12, 23, 123$) indicates the configuration after the merger. For example, σ_{12} is the configuration in which partitions 1 and 2 are under a single government, Town 12. In addition, we focus only on the case where towns under the configuration σ_ϕ choose whether to merge. In other words, we set the configuration σ_ϕ as the initial state.

<<Table 2: About HERE>>

In the process of the merger, we consider the following three alternative rules, and evaluate the welfare effects of rule setting:

i) Majority Voting: each town is free to choose whether to merge with its counterparts;

ii) Municipal Request: merging of towns takes place once a single town requests it;

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represented by included partition and the distance to the facility. Hence, without the initial geographical distribution of households, each type's residence becomes indeterminate, and we assume that households are geographically aligned according to their preference toward the public good. However, we conduct the similar analysis and obtain the qualitatively similar results when this restrictive assumption is relaxed by allowing that geographical distribution of households does not coincide with the one in the preference space.

iii) Central Approval: the central government chooses whether to implement the political merger of towns.

2.2 Households

All households attain the utility from the consumption of private and public goods, z and g respectively. Suppose that a type θ household reside in Town i under the configuration σ;[5] then, the utility of type θ in Town i is given by

$$U\left(z_i, g_i; \theta, \sigma\right) = z_i + \theta \ln g_i. \tag{1}$$

Each household is endowed with w units of the private good, which is treated as a numeraire. In addition, according to the public ownership of land, the aggregate land rents, R, are equally redistributed among all households. Therefore, households use $w + R / N$ units of the private good for the head tax levied in residing Town i, τ_i, the land rent at their residence x, $r_i(x)$, and the access cost from their residence to the facility x_i, $t\,|\,x - x_i\,|$ where t is the access cost per distance. The residual of the private good, z_i, is self-consumed. Therefore, the budget constraint for a household in Town i is:

$$z_i = w + \frac{R}{N} - \tau_i - t\left|x - x_i\right| - r_i\left(x\right). \tag{2}$$

Instead of solving the utility maximization problem of each type θ directly, we solve this by using the spatial arbitrage with in a town. Let us denote by $\overline{r_i}$ the land

[5] The residence of a type θ household, i, is automatically determined once the configuration σ is set. That is, formally, each type's residence i is written as the function of σ: $i = h(\sigma; \theta)$.

rent at the location furthest from the public facility (hereafter, we call this location the periphery). In addition, let us define by $b_i(x_i)$, the maximal distance from the public facility in Town i, then this suffices the following:

$$b_i\left(L_i^m\right)=\frac{L_i}{2}\leq b_i\left(x\right)\leq b_i\left(L_k^b\right)=b_i\left(L_i^b\right)=L_i \text{ for } x\in\left[L_k^b,L_i^b\right], \tag{3}$$

where L_i^m, L_k^b, and L_i^b are the center, and the left and the right boundaries of this town.

As shown in Eq. (1), residents in Town i consume the same amount of the private good, z_i. Therefore, given the location of the public facility, x_i, Eq. (2) is rewritten as:

$$z_i=w+\frac{R}{N}-\tau_i-t\left|x-x_i\right|-r_i\left(x\right)=w+\frac{R}{N}-\tau_i-r_i\left(x_i\right)=w+\frac{R}{N}-\tau_i-tb_i\left(x_i\right)-\overline{r}_i.$$

Hence, the sum of the land rent and the access cost at the location x is computed as:

$$r_i\left(x\right)+t\left|x-x_i\right|=\overline{r}_i+tb_i\left(x_i\right).$$

By using this, the self-consumed private goods in Town i is:

$$z_i\left(x_i,\tau_i,R_i\right)=w+R_i-\tau_i-tb_i\left(x_i\right), \tag{4}$$

where R_i is the net income from the land defined as:

$$R_i\equiv\frac{R}{N}-\overline{r}_i.$$

Plugging Eq. (4) into Eq. (1) and rearranging, the utility of type θ in Town i is computed as:

$$u\left(x_i,\tau_i,g_i,R_i;\theta,\sigma\right)=U\left(z_i\left(x_i,\tau_i,R_i\right),g_i;\theta,\sigma\right)=w+R_i-\tau_i-tb_i\left(x_i\right)+\theta\ln g_i. \tag{5}$$

2.3 Local Governments

The technology of public good production is identical among towns: c units of the private good are used for a unit of the public good production. Specifically, costs of public good production, $C(g_i)$, are:

$$C(g_i) = cg_i.$$

The costs of the public good production are financed by the head tax revenue. When we represent by N_i the population of Town i, the balanced budget constraint for the local government of that town is:

$$\tau_i = \frac{cg_i}{N_i}. \tag{6}$$

Since the location of the public facility, x_i, and the amount of the public good, g_i, is determined by a referendum, the problem of Town i government under the configuration σ is formulated as the utility maximization of the median voter, $\theta_i^m = (\theta_{i-1}^b + \theta_i^b)/2$.[6] That is,

$$\max_{g_i, \tau_i, x_i} u\left(x_i, \tau_i, g_i, R_i; \theta_i^m, \sigma\right) \text{ subject to (3) and (6).} \tag{7}$$

First, notice that the utility of the median voter is maximized when the maximal distance $b_i(x_i)$ is minimized. Therefore, according to Eq. (3), the government chooses the location of the facility at the middle of its jurisdiction, $x_i^*(\sigma) = L_i^m = (L_{i-1}^b + L_i^b)/2$.

[6] Since we have assumed the uniform distribution of θ, the average preference intensity within a town coincides with the median within that town.

Second, by plugging Eq. (6) into the objective function of (7) and differentiating it with respect to g_i,

$$-\frac{c}{N_i}+\frac{\theta_i^m}{g_i}=0. \tag{8}$$

According to Equations (6) and (8), we obtain the amount of the public good and the head tax in Town *i* as follows:

$$g_i^*(\sigma)=\frac{\theta_i^m N_i}{c} \text{ and } \tau_i^*(\sigma)=\theta_i^m. \tag{9}$$

2.4 Residential Equilibrium

Finally, we describe the residential choice of the boundary types, θ_i^b who resides in two neighboring towns. Substituting Eq. (9) and $x_i = L_i^m$ and $b(L_i^m) = L_i/2$ into Eq. (5), the utility of type θ in Town *i*, is rewritten as:

$$v(\theta,\sigma,R_i)=u\left(x_i^*(\sigma),\tau_i^*(\sigma),g_i^*(\sigma),R_i;\theta,\sigma\right)=w+R_i-\theta_i^m-\frac{tL_i}{2}+\theta\ln\frac{\theta_i^m N_i}{c}. \tag{10}$$

In each town, at least one of two boundary types of Town *i* also resides in its neighboring town. At the equilibrium, these types should be indifferent to residing in these towns, and we assume that the land rents at the periphery, $\overline{r}_i$, compensate for the difference in the utilities of the boundary type between two neighboring towns.

Suppose that, under the configuration σ, Towns *i* and *k* are neighboring and type θ_i^b resides in these two towns. In such case, the following must hold for this type:

$$v\left(\theta_i^b,\sigma,R_i\right)\equiv v\left(\theta_i^b,\sigma,R/N-\overline{r}_i\right)=v\left(\theta_i^b,\sigma,R/N-\overline{r}_k\right)\equiv v\left(\theta_i^b,\sigma,R_k\right). \tag{11}$$

By solving (11) with respect to $\overline{r}_k$,

$$\overline{r}_k = \overline{r}_i + \frac{t\left(N_i - N_k\right)}{2} + \left(\theta_i^m - \theta_k^m\right) + \theta_i^b \ln \frac{\theta_k^m N_k}{\theta_i^m N_i}. \qquad (12)$$

In Eq. (12), the second term of the RHS captures the difference in the maximal access cost to the public facility, and under σ_ϕ, it is equal to zero. The third and fourth terms respectively captures the difference in the head tax and the benefit of the public good.

In case of the configuration σ_ϕ, the land rent at the location *x* is given by:

$$r\left(x, \overline{r}_1; \sigma_\phi\right) = \begin{cases} \overline{r}_1 + \dfrac{t\overline{N}}{2} - t\left|x - x_1\right| \\ \overline{r}_1 + \dfrac{t\overline{N}}{2} - t\left|x - x_i\right| + \displaystyle\sum_{k=2}^{i}\left[\left(\theta_{k-1}^m - \theta_k^m\right) + \theta_{k-1}^b \ln \frac{\theta_k^m}{\theta_{k-1}^m}\right] \text{ for } i = 2,3. \end{cases} \qquad (13)$$

Let us denote by $R_i(\sigma_\phi)$ the net income from land for Town *i* residents: then by using Eq. (12), this is computed as follows:

$$\begin{aligned} R_i\left(\sigma_\phi\right) &\equiv \frac{R}{N} - \overline{r}_i\left(\sigma_\phi; \overline{r}_1\right) = \frac{1}{N}\int_0^L r_i\left(x, \overline{r}_1; \sigma_\phi\right)dx - \overline{r}_i\left(\sigma_\phi, \overline{r}_1\right) \\ &= \frac{1}{3}\left[\sum_{k=i}^{3}(3-k)\left[\left(\theta_{k-1}^m - \theta_k^m\right) + \theta_k^b \ln \frac{\theta_k^m}{\theta_{k-1}^m}\right] - \sum_{k=1}^{i}(k-1)\left[\left(\theta_k^m - \theta_{k+1}^m\right) + \theta_k^b \ln \frac{\theta_{k+1}^m}{\theta_k^m}\right]\right] + \frac{t\overline{N}}{4}, \end{aligned}$$

(14)

where

$$\overline{r}_i\left(\sigma_\phi, \overline{r}_1\right) = \begin{cases} \overline{r}_1 \\ \overline{r}_1 + \displaystyle\sum_{k=2}^{i}\left[\left(\theta_{k-1}^m - \theta_k^m\right) + \theta_{k-1}^b \ln \frac{\theta_k^m}{\theta_{k-1}^m}\right] \text{ for } i = 2,3. \end{cases}$$

In comparison of Eq. (14), we obtain the following lemma:

Lemma 1

Under configuration σ_{ϕ}, *residents in Town 1 enjoy positive net land income while those in Town 3 incur negative net land income.*

Lemma 1 tells us that, due to the capitalization of public good benefits, residents in Town 3 pay more for their residence, and receive less from lands owned in other towns. In contrast, residents in Town 1 gain from the public goods provided in Towns 2 and 3, which increase the payment for their owned land in these two towns. To put it differently, the capitalization of public good benefits on the land rent generates the pecuniary externality from the town providing the public good more to the one providing less.

Following a process similar to the configuration σ_{ϕ}, the net land income under other configuration (σ_{12}, σ_{23}, and σ_{123}) is computed.[7] Plugging these net incomes, $R_i(\sigma)$, into Eq. (10), the utility of a type θ household under the configuration σ is computed as $V(\theta,\sigma) = U(\theta,\sigma,R_i(\sigma))$: specifically,

$$V\left(\theta,\sigma_{\phi}\right) = w + R_i\left(\sigma_{\phi}\right) - \theta_i^m - \frac{t\bar{N}}{2} + \theta \ln \frac{\theta_i^m \bar{N}}{c}, \text{ for } \theta \in \left[\theta_{i-1}^b, \theta_i^b\right], \qquad (15\text{-}1)$$

$$V\left(\theta,\sigma_{12}\right) = \begin{cases} w + R_{12}\left(\sigma_{12}\right) - \theta_{12}^m - t\bar{N} + \theta \ln \dfrac{2\theta_{12}^m \bar{N}}{c}, \text{ for } \theta \in \left[\theta_0^b, \theta_2^b\right], \\ w + R_3\left(\sigma_{12}\right) - \theta_3^m - \dfrac{t\bar{N}}{2} + \theta \ln \dfrac{\theta_3^m \bar{N}}{c}, \text{ for } \theta \in \left[\theta_2^b, \theta_3^b\right], \end{cases} \qquad (15\text{-}2)$$

[7] See Appendix A.

$$V(\theta,\sigma_{23})=\begin{cases} w+R_1(\sigma_{23})-\theta_1^m-\dfrac{t\bar{N}}{2}+\theta\ln\dfrac{\theta_1^m\bar{N}}{c}, \text{ for } \theta\in\left[\theta_0^b,\theta_1^b\right], \\ w+R_{23}(\sigma_{23})-\theta_{23}^m-t\bar{N}+\theta\ln\dfrac{2\theta_{23}^m\bar{N}}{c}, \text{ for } \theta\in\left[\theta_1^b,\theta_3^b\right], \end{cases} \tag{15-3}$$

$$V(\theta,\sigma_{123})=w-\theta_{123}^m-\frac{3t\bar{N}}{4}+\theta\ln\frac{3\theta_{123}^m\bar{N}}{c}, \text{ for } {}^\forall\theta. \tag{15-4}$$

Note that, under the configuration σ_{123}, no residents receive the net land income since the pecuniary spillover is fully internalized. By using Equations (15), the social welfare is computed as:

$$SW(\sigma)=\int_{\underline{\theta}}^{\bar{\theta}}V(\theta,\sigma)\,d\theta \text{ for } \sigma=\sigma_\phi,\sigma_{12},\sigma_{23},\sigma_{123}. \tag{16}$$

3. The Merger under Alternative Rules

This section investigates the merger under three alternative rules, such as i) Majority Voting; ii) Municipal Request; iii) Central Approval. Under Rule iii), the central government chooses its configuration according to the referendum within the entire economy. In contrast, under Rules i) and ii), the choices of local governments lead to the configuration change. Rule i) requests that towns choose whether to merge with its neighbor according to the referendum within its jurisdiction. The merger is realized when it is accepted by the majority of residents in each town joining to. In case of Rule ii), each town implements the referendum for whether to change the political configuration. Once the configuration change is approved in one of towns, this town requests for altering the configuration, and it is immediately accepted by the central government.

This section is organized as follows; in Subsection 3.1, we focus on the choices of local governments on the political merger. Subsection 3.2 formulates the problem under Rule i), and summarizes how the political configuration is determined under this rule. Subsection 3.3 shows the political configuration under Rule ii) while Subsection 3.4 formulates the problem of the central government, and shows the results under Rule iii).

3.1 Choice of Local Governments

Since the referendum for the configuration is introduced in each town under Rules i) and ii), Town *i* determines their choice in order to maximize the utility of their median voter, θ_i^m. Note that, however, since the political configuration, σ, is determined by the outcome of three towns' choices, $\mathbf{s} = (s_1, s_2, s_3)$, it is expressed as the function of **s**: namely $\sigma = \sigma(\mathbf{s})$. When the referendum for the merger is introduced in each town, Town *i* (*i*=1, 2, 3) chooses the strategy, s_i, in order to maximize the median voter's utility. If Town *i* wants to merge with its neighbor *j*, then $s_i=\{i, j\}$; if this Town *i* wants to keep the isolation, then $s_i=\{i\}$. Formally, let us denote by s_i^* the strategy chosen by Town *i*, then it satisfies:

$$s_i^* = \arg\max_{s_i} V\left(\theta_i^m, \sigma\left(\mathbf{s}\right)\right). \tag{17}$$

Let us first compare the payoffs under the two-town configurations, σ_{12} and σ_{23}. According to the comparison, we obtain the following lemma:

Lemma 2

All three towns under the configuration σ_{ϕ} *prefer the configuration* σ_{23} *to* σ_{12}.

Proof: see Appendix C.

This result stems from the heterogeneity of residents among three towns. Taking the differences in payoffs of three towns between configurations σ_{23} and σ_{12}:

$$V\left(\theta_1^m,\sigma_{23}\right)-V\left(\theta_1^m,\sigma_{12}\right)=\theta_1^m \ln\frac{\theta_1^m}{2\theta_{12}^m}+\frac{1}{3}\left(2\theta_1^b \ln\frac{2\theta_{23}^m}{\theta_1^m}+\theta_2^b \ln\frac{2\theta_{12}^m}{\theta_3^m}\right)>0, \quad (18\text{-}1)$$

$$V\left(\theta_2^m,\sigma_{23}\right)-V\left(\theta_2^m,\sigma_{12}\right)=\theta_2^m \ln\frac{\theta_{23}^m}{\theta_{12}^m}+\frac{1}{3}\left(\theta_1^b \ln\frac{\theta_1^m}{2\theta_{23}^m}+\theta_2^b \ln\frac{2\theta_{12}^m}{\theta_3^m}\right)>0, \quad (18\text{-}2)$$

$$V\left(\theta_3^m,\sigma_{23}\right)-V\left(\theta_3^m,\sigma_{12}\right)=\theta_3^m \ln\frac{2\theta_{23}^m}{\theta_3^m}+\frac{1}{3}\left(\theta_1^b \ln\frac{\theta_1^m}{2\theta_{23}^m}+2\theta_2^b \ln\frac{\theta_3^m}{2\theta_{12}^m}\right)>0. \quad (18\text{-}3)$$

In each of Equations (18), the first term of the LHS captures the difference in the benefit from the public good between two configurations σ_{23} and σ_{12} while the second parentesis term is the difference in the net land income. For residents in Town 3, the merger with Town 2 results in the increase in the public good provision and the decrease in the net land income due to the capitalization of the scale economy on the their residence. According to Lemma 2, however, residents in Town 3 outweigh the scale economy against the negative impact of the merger on their net land income. For residents in Town 1, allowing the merger of Towns 2 and 3 increases the net land income while it leads to the smaller amount of the public good provision compared to merging with Town 2. Lemma 2 indicates that since the rise in the net income exceeds the loss in the public good benefit, Town 1 allows the merger of Towns 2 and 3, and keeps its isolation. In contrast to Towns 1 and 3, residents in Town 2 have to choose its counterpart of the merger. In either configuration, they experience the fall in the income while the public good provision differs with the counterpart of the merger. According to Lemma 2, since residents in Town 2 can enjoy the more amounts of the public good by merging with Town 3, they choose Town 3 as their counterpart of the merger.

By using Lemma 2, we can limit our focus on three configurations out of four such as σ_{ϕ}, σ_{23}, and σ_{123}. Let us now deal with the problem how the parameter value, affect each town's preference toward the merger or the configuration. According to the comparison of each town's payoffs between the configurations σ_y and $\sigma_{y'}$ we can derive the threshold of the access cost $t = t_i(\sigma_y, \sigma_{y'})$ at which

$$V\left(\theta_i^m, \sigma_y\right) = V\left(\theta_i^m, \sigma_{y'}\right).$$

For $t > t_i(\sigma_y, \sigma_{y'})$, Town i prefers the configuration $\sigma_{y'}$ to σ_y ($V(\theta_i^m, \sigma_y) < V(\theta_i^m, \sigma_{y'})$); otherwise, the configuration σ_y is preferred ($V(\theta_i^m, \sigma_y) > V(\theta_i^m, \sigma_{y'})$).

By comparing thresholds $t_i(\sigma_y, \sigma_{y'})$ for each Town *i*, we obtain the strategy of Town *i*, s_i^*, as in Lemma 3:

Lemma 3

i) Towns 2 and 3 choose the isolation, $s_i^* = \{i\}$, *if* $t > t_i(\sigma_{23}, \sigma_\phi)$ *while they support the merger of the three towns,* $s_i^* = \{1, 2, 3\}$ *if* $t_i(\sigma_{123}, \sigma_{23}) \geq t$ *. When* $t_i(\sigma_{23}, \sigma_\phi) \geq t > t_i(\sigma_{123}, \sigma_{23})$, *they prefer a two-town merger.*

ii) Town 1 prefers isolation, $s_1^* = \{1\}$, *if* $t > t_i(\sigma_{123}, \sigma_{23})$ *; otherwise, it chooses the three-town merger* $s_1^* = \{1, 2, 3\}$.

Proof: see Appendix B.

Lemma 3 solely states that, as access costs for the public facilities decrease, each town becomes more willing to merge with its neighbors.

Finally we compare the thresholds, $t_i(\sigma_y, \sigma_{y'})$, among three towns, and this result is summarized in Lemma 4:

Lemma 4

Among three towns, Town 1's threshold, $t_1(\sigma_y, \sigma_{y'})$, *is the lowest while Town 3's,* $t_3(\sigma_y, \sigma_{y'})$, *is the highest for any pair of configurations,* σ_y *and* $\sigma_{y'}$.

Proof: see Appendix B.

This lemma states that the preference toward the configuration change differs among towns. Namely, residents in Town 3 can sacrifice more access cost to achieve the configuration change while a small increase in the access cost may make those in Town 1 reject configuration change. Since residents in Town 3 are the major beneficiaries of the scale economy from a merger, they are most willing to merge with its neighbors. In contrast, residents in Town 1 are the major beneficiaries of the pecuniary externality through land rent, which diminishes through the merger; therefore, they will be most inclined to oppose a the merger.

3.2 The Majority Voting

Under this rule, the merger takes place if relevant towns agree. Since the configuration is determined via negotiation among towns, each town's choice s_i^* does not necessarily coincide with the outcome of Majority Voting. Therefore, it is necessary to define the equilibrium for Majority Voting. Let us represent by $f(\sigma_y)$ the set of towns merged under the configuration σ_y; then, under the configuration σ_{23}, $f(\sigma_{23}) = \{2,3\}$ while under the configuration σ_ϕ, $f(\sigma_\phi) = \phi$. By using $f(\sigma_y)$ and s_i^* in Eq. (17), the following definition summarizes the equilibrium of the Majority Voting:[8]

Definition

[8] This definition is based on the Delta Stability in Hart and Kurz (1983). Jackson and Wolinsky (1996) also study the negotiation among players and formulate the Pairwise Stability. However, it is not inapplicable in our model since the pairwise stability is based on the negotiation between two players.

The configuration σ_y *is the equilibrium configuration of the Majority Voting if the following two conditions are satisfied:*

$$\bigcap_{i\in f(\sigma_y)} s_i^* = f\left(\sigma_y\right), \tag{19-1}$$

$$k \notin s_k^* \cap \bigcap_{i\in f(\sigma_y)} s_i^*, \text{ for } k \notin f\left(\sigma_y\right). \tag{19-2}$$

In order to make image of this definition clear, let us focus on the configuration σ_{23}. Condition (19-1) requests that σ_{23} is the equilibrium if $s_3^* = \{1,2,3\}$ and $s_2^* = \{2,3\}$ as well as $s_2^* = s_3^* = \{2,3\}$. Furthermore, condition (19-2) urges that the common set of all three towns' strategies does not include Town 1. In other words, to assure the merger of Towns 2 and 3 as the equilibrium, either Town 1 must choose to keep the isolation, or Towns 2 and 3 must reject the participation of Town 1 in the merger.

As shown in Subsection 4.1, each town has a different value of the threshold for a configuration change from $\sigma_{y'}$ to σ_y. Given the definition of $t_i(\sigma_y, \sigma_{y'})$, among towns merged under the configuration σ_y, the town with the lowest threshold is decisive in the negotiation of the merger. Denoting the minimum threshold among towns merged under σ_y by $t^E(\sigma_y, \sigma_{y'})$ implies

$$t^E\left(\sigma_y, \sigma_{y'}\right) = \min_{i\in f(\sigma_y)} t_i\left(\sigma_y, \sigma_{y'}\right). \tag{20}$$

Then for $t < t^E(\sigma_y, \sigma_{y'})$, all towns participating in the merger under σ_y can improve their median voters' welfares by the merger; otherwise, the merger harms the welfare of at least one of towns. By using the Definition and $t^E(\sigma_y, \sigma_{y'})$, we can derive the equilibrium configuration of the Majority Voting, σ^E:

<<Figure 2: About HERE>>

Proposition 1

The configuration σ_ϕ *is the equilibrium for the case of Majority Voting if* $t > t^E(\sigma_{23}, \sigma_\phi)$ *; the configuration* σ_{23} *, if* $t^E(\sigma_{23}, \sigma_\phi) \geq t > t^E(\sigma_{123}, \sigma_{23})$ *; the configuration* σ_{23} *, if* $t^E(\sigma_{123}, \sigma_\phi) \geq t$*. Formally,*

$$\sigma^E = \begin{cases} \sigma_\phi \text{ for } t > t^E\left(\sigma_{23}, \sigma_\phi\right), \\ \sigma_{23} \text{ for } t^E\left(\sigma_{23}, \sigma_\phi\right) \geq t > t^E\left(\sigma_{123}, \sigma_{23}\right), \\ \sigma_{123} \text{ for } t^E\left(\sigma_{123}, \sigma_{23}\right) \geq t. \end{cases}$$

Proof: see Appendix B.

Figure 2 plots thresholds in Proposition 1 on the (α, t) space. This figure shows that as the access cost, *t*, increases, the merger via Majority Voting becomes harder to achieve. Furthermore, for sufficiently large value of α, a three-town merger via Majority Voting cannot be achieved . This is because, for residents in Town 1, the loss of the net land income is too large when the degree of the heterogeneity among households is sufficiently large; consequently, they refuse to merge with other two towns.

4.3 The Municipal Request

Under this rule, the change of the configuration takes place according to the following procedure. First, each of three towns sets out a referendum for the configuration change. Once the configuration change is approved in one of three towns, it is always realized. Since, as in Lemma 4, residents in Town 3 are more willing to merge with others, Town 3 always requests configuration change. In other words, under this rule, the threshold for the configuration change is equal to one for Town 3: that is, if we express by $t^R(\sigma_y, \sigma_{y'})$ the threshold under this rule, $t^R(\sigma_y, \sigma_{y'}) = t_3(\sigma_y, \sigma_{y'})$. By using this relation, the configuration under the Municipal Request, σ^R, is derived as:

Proposition 2

Under the Municipal Request, Town 3 always requests a merger so that the configuration σ_ϕ *is the outcome if* $t > t^R(\sigma_{23}, \sigma_\phi)$ *; the configuration* σ_{23} *, if* $t^R(\sigma_{23}, \sigma_\phi) \geq t > t^R(\sigma_{123}, \sigma_{23})$ *; the configuration* σ_{123}*, if* $t^R(\sigma_{123}, \sigma_{23}) \geq t$ *. Formally,*

$$\sigma^R = \begin{cases} \sigma_\phi \text{ for } t > t^R\left(\sigma_{23}, \sigma_\phi\right), \\ \sigma_{23} \text{ for } t^R\left(\sigma_{23}, \sigma_\phi\right) \geq t > t^R\left(\sigma_{123}, \sigma_{23}\right), \\ \sigma_{123} \text{ for } t^R\left(\sigma_{123}, \sigma_{23}\right) \geq t. \end{cases}$$

Proof: Since $t^R(\sigma_y, \sigma_{y'}) = t_3(\sigma_y, \sigma_{y'})$, it is obvious from Lemma 3.

QED

This result is qualitatively similar to the equilibrium of the Majority Voting, σ^E: namely, as the access cost, *t*, increases, the configuration change becomes more difficult to be realized. Also note that, reconfiguration by Municipal Request is always easier than Majority Voting since, as in Lemma 4, $t^R(\sigma_y, \sigma_{y'}) = t_3(\sigma_y, \sigma_{y'}) > t^E(\sigma_y, \sigma_{y'})$.

4.4 The Central Approval

In this case, the central government seeks referendum for the configuration change accross the entire economy. The configuration change is implemented once it is supported by the majority of the economy. To put it differently, the central government solves the following problem:

$$\max_{\sigma} V\left(\theta^M, \sigma\right),$$

where $\theta^M = (\overline{\theta} + \underline{\theta})/2$ is the median voter of the entire economy. In our setting, however, the median voter of the economy is identical to that of Town 2; that is, $\theta^M = \theta_2^m$. Therefore, if we denote by $t^C(\sigma_y, \sigma_{y'})$ the threshold under this rule, $t^C(\sigma_y, \sigma_{y'}) = t_2(\sigma_y, \sigma_{y'})$.[9] By using Lemma 3, the configuration under the Central Approval, σ^C, is derived as:

Proposition 3

Under the Central Approval, the configuration σ_ϕ *is the outcome if* $t > t^C(\sigma_{23}, \sigma_\phi)$*;*

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[9] By following the similar procedure described in Subsection 3.1, we obtain the threshold $t = t^C(\sigma_y, \sigma_{y'})$ at which:

$$V\left(\theta^M, \sigma_y\right) = V\left(\theta^M, \sigma_{y'}\right).$$

In addition, for $t < t^C(\sigma_y, \sigma_{y'})$, type θ^M households prefer the configuration σ_y to $\sigma_{y'}$; otherwise, $\sigma_{y'}$ is preferred.

the configuration σ_{23} *, if* $t^C(\sigma_{23},\sigma_{\phi}) \geq t > t^C(\sigma_{123},\sigma_{23})$ *; the configuration* σ_{23} *, if* $t_2(\sigma_{123},\sigma_{23}) \geq t$ *. Formally,*

$$\sigma^C = \begin{cases} \sigma_{\phi} \text{ for } t > t_2\left(\sigma_{23},\sigma_{\phi}\right), \\ \sigma_{23} \text{ for } t_2\left(\sigma_{23},\sigma_{\phi}\right) \geq t > t_2\left(\sigma_{123},\sigma_{23}\right), \\ \sigma_{123} \text{ for } t_2\left(\sigma_{123},\sigma_{23}\right) \geq t. \end{cases}$$

The configuration under this rule, σ^C, also qualitatively resembles to those under other settings.

4. The Efficiency of Alternative Rules for the Political Merger

In order to evaluate the efficiency of the alternative rules on the political merger, in Subsection 4.1, we investigate the configuration which maximizes the social welfare. Hereafter, we set this configuration as the benchmark for the evaluation of alternative rules, and we call it the welfare-maximizing configuration. In Subsection 4.2, by comparing the configurations under three alternative rules, σ^E, σ^R, and σ^C with the welfare-maximizing, we evaluate the welfare effect of alternative rules.

4.1. The Welfare-Maximizing Configuration

This subsection derives the welfare-maximizing configuration, which is treated as the benchmark for the evaluation of alternative rules. In deriving this configuration, we assume that the boundaries of three partitions are exogenously given: that is, no towns are formed by dividing three partitions. In this sense, the welfare-maximizing configuration deviates from the optimum. Formally, this configuration is derived according to the following problem:

$$\max_{\sigma} SW(\sigma). \tag{21}$$

In the process of solving the problem (21), we compute $t = t^O(\sigma_y, \sigma_{y'})$ according to the following relation:

$$SW(\sigma_y) = SW(\sigma_{y'}).$$

Furthermore, for $t > t^O(\sigma_y, \sigma_{y'})$, we have $SW(\sigma_y) < SW(\sigma_{y'})$; otherwise, $SW(\sigma_y) > SW(\sigma_{y'})$. In addition, it is easily show that the thresholds, $t^O(\sigma_y, \sigma_{y'})$, are solely dependent on the degree of the heterogeneity, $\alpha \equiv \overline{\theta} / \underline{\theta}$.[10] By using this threshold, Proposition 4 summarizes the welfare-maximizing configuration σ^O.

Proposition 4

The configuration σ_ϕ *maximizes the social surplus if* $t > t^O(\sigma_{23}, \sigma_\phi)$ *; the configuration* σ_{23} *, if* $t^O(\sigma_{23}, \sigma_\phi) \geq t > t^O(\sigma_{123}, \sigma_{23})$ *; the configuration* σ_{123} *, if* $t^O(\sigma_{123}, \sigma_{23}) \geq t$ *. Formally,*

$$\sigma^O = \begin{cases} \sigma_\phi \text{ for } t > t^O(\sigma_{23}, \sigma_\phi), \\ \sigma_{23} \text{ for } t^O(\sigma_{23}, \sigma_\phi) \geq t > t^O(\sigma_{123}, \sigma_{23}), \\ \sigma_{123} \text{ for } t^O(\sigma_{123}, \sigma_{23}) \geq t. \end{cases}$$

Proof: see Appendix C.

[10] See Appendix C.

<<Figure 3: About HERE>>

Figure 3 plots thresholds in Proposition 4 on the (α, t) space, and it shows that as in the outcomes summarized in Section 3, the configuration changes from the three-town, σ_{ϕ}, to the single-town, σ_{123} via the two-town, σ_{23} as the access cost, *t*, decreases. This result indicates that the political merger is socially desired when the access cost is sufficiently low. Furthermore, as in the outcomes under three alternative rules, between the two-town configurations, σ_{12} and σ_{23}, the configuration σ_{12} is never realized as the welfare-maximizing. This is because, since the residents in Town 3 have relatively strong preference intensity toward the public good compared to those in Town 1, it is more efficient to introduce the scale economy in Town 3 through the merger.

4.2 The Evaluation of Alternative Rules for the Political Merger

This subsection addresses the evaluation of alternative three rules described in Section 3 by comparing with the welfare-maximizing configuration. Prior to investigate the welfare effects of three rules, we compare the thresholds, $t^T(\sigma_y, \sigma_{y'})$ (*T* = *E*, *R*, *C*, *O*):

Proposition 5

The threshold $t^T(\sigma_y, \sigma_{y'})$ *has the following relationship:*

$$t^E\left(\sigma_{123}, \sigma_{23}\right) < t^O\left(\sigma_{123}, \sigma_{23}\right) < t^C\left(\sigma_{123}, \sigma_{23}\right) < t^R\left(\sigma_{123}, \sigma_{23}\right), \tag{22-1}$$

$$t^{E}\left(\sigma_{23},\sigma_{\phi}\right)=t^{C}\left(\sigma_{23},\sigma_{\phi}\right)<t^{O}\left(\sigma_{23},\sigma_{\phi}\right)<t^{R}\left(\sigma_{23},\sigma_{\phi}\right). \tag{22-2}$$

Proof: see Appendix D.

QED

Eq. (22-1) summarizes the relationship of thresholds for three-town merger while Eq. (22-2) compares the thresholds for the merger of Towns 2 and 3. In case of three-town merger, the decisive town for the configuration varies with rule: that is, Town 1 under the Majority Voting; Town 2 under the Central Approval; Town 3 under the Municipal Request. Therefore, by computing the difference in the payoffs of three towns between two configurations, σ_{23} and σ_{123}, we can approach to the difference in outcomes among three rules:

$$V\left(\theta_1^m,\sigma_{123}\right)-V\left(\theta_1^m,\sigma_{23}\right)=\theta_1^m \ln\frac{3\theta_{123}^m}{\theta_1^m}-\frac{2\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_1^m}-\frac{t\bar{N}}{3}, \tag{23-1}$$

$$V\left(\theta_i^m,\sigma_{123}\right)-V\left(\theta_i^m,\sigma_{23}\right)=\theta_i^m \ln\frac{3\theta_{123}^m}{2\theta_{23}^m}+\frac{\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_1^m}-\frac{t\bar{N}}{3} \text{ for } i=2,3. \tag{23-2}$$

In both Equations, the first term of the RHS is the benefit of the merger, the increase in the public good through the scale economy; the second term is the change in the net land income; and the third term is the increase in the access cost. For residents in Town 1, the merger results in the loss of the positive externality from the Town 23 through the net land income; therefore, they become less willing to merge with Town 23. In contrast, under the configuration σ_{23}, Town 23 is the source of the pecuniary externality;

therefore, they can reduce their payment for the residence by merging with Town 1. Consequently, they are more willing to merge than is Town 1. In other words, these two towns have incentives to internalize the pecuniary externality by choosing the merger.

Evaluating Equations (23-1) and (23-2) at $t = t^O(\sigma_{123}, \sigma_{23})$:

$$V\left(\theta_1^m, \sigma_{123}\right) - V\left(\theta_1^m, \sigma_{23}\right) = \left[\theta_1^m \ln\frac{3\theta_{123}^m}{\theta_1^m} - \frac{2\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_1^m}\right] - \frac{1}{3}\left[\theta_1^m \ln\frac{3\theta_{123}^m}{\theta_1^m} + \sum_{i=2,3}\theta_i^m \ln\frac{3\theta_{123}^m}{2\theta_{23}^m}\right] < 0,$$

$$V\left(\theta_i^m, \sigma_{123}\right) - V\left(\theta_i^m, \sigma_{23}\right) = \left[\theta_i^m \ln\frac{3\theta_{123}^m}{2\theta_{23}^m} + \frac{\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_1^m}\right] - \frac{1}{3}\left[\theta_1^m \ln\frac{3\theta_{123}^m}{\theta_1^m} + \sum_{i=2,3}\theta_i^m \ln\frac{3\theta_{123}^m}{2\theta_{23}^m}\right] > 0 \text{ for } i = 2,3.$$

In these equations, the first bracket term captures the net benefit of the merger for each town while the second bracket term is the average net benefit for the economy. These two equations show that, due to the pecuniary externality through the land rent, Town 1 underestimate their net benefit of the merger while Towns 2 and 3 overestimate theirs compared to the net benefit of the entire economy. The similar mechanism is realized under the case of the merger of Towns 2 and 3.[11]

Because of the capitalization, the Municipal Request makes the merger easier while the Majority Voting makes it harder compared to the welfare-maximizing level.

[11] In this case, Town 2 is pivotal in determining configuration under the Majority Voting and the Central Approval. As in Lemma 1, the merger with Town 3 implies that Town 2 gives up the net land income. Consequently, Town 2 becomes defensive against the merger. In contrast, residents in Town 3 can reduce the payment for their residence through the merger; therefore, they become more aggressive against the merger.

Therefore, we need to deal with the problem such that under what circumstances, which type of rules should be implemented. In order to solve this problem, we denote by SW^T the social welfare under the configuration σ^T:

$$SW^T = SW\left(\sigma^T\right) \text{ for } T = E, R, C.$$

By using this, Proposition 6 compares the social welfares under three alternative rules:

Proposition 6

The social welfare SW^T ($T=E, R, C$) has the following relationship:

$$SW^R = SW^C \geq SW^E \text{ for } t^E\left(\sigma_{123}, \sigma_{23}\right) < t \leq t^O\left(\sigma_{123}, \sigma_{23}\right),$$

$$SW^E > SW^R = SW^C \text{ for } t^O\left(\sigma_{123}, \sigma_{23}\right) < t < t^C\left(\sigma_{123}, \sigma_{23}\right),$$

$$SW^E = SW^C \geq SW^R$$
$$\text{for } t^C\left(\sigma_{123}, \sigma_{23}\right) \leq t \leq t^R\left(\sigma_{123}, \sigma_{23}\right)$$
$$\text{and } t^O\left(\sigma_{23}, \sigma_{\phi}\right) \leq t \leq t^R\left(\sigma_{23}, \sigma_{\phi}\right),$$

$$SW^R > SW^E = SW^C \text{ for } t^E\left(\sigma_{23}, \sigma_{\phi}\right) < t < t^O\left(\sigma_{23}, \sigma_{\phi}\right),$$

Proof: It is obvious from the definition of $t^O(\sigma_y, \sigma_{y'})$ and Proposition 5.

QED

Figure 4 summarizes Proposition 6 on the (α, t) space, and it shows that different type of rules becomes efficient under the different sets of parameter values. For example, in case of the three-town merger, when both the access cost is relatively low against the degree of the heterogeneity (point A of Figure 4), the Municipal Request or the Central

Approval replicates the welfare-maximizing configuration. In contrast, when the access cost is relatively high (point B of Figure 4), the Majority Voting or the Central Approval achieves the same configuration as the welfare-maximizing one.

<<Figure 4: About HERE>>

In reality, there exists a global tendency toward the reorganization of local governments, especially in metropolitan areas, and this type of the reorganization aims at internalizing the spillover of the benefit of public goods provided in the central city. This situation corresponds to the case of the merger of Towns 1 and 23 in our model. Furthermore, the access cost becomes lower than before due to the technology progress, and the tastes of households become more heterogeneous. Even in such a situation, however, our result suggests that it is ambiguous to tell which type of the rule setting replicates the welfare-maximizing outcome. If the magnitude of the access cost, t, is relatively large compared to the degree of the heterogeneity, α, the decentralized negotiation will be the second-best rule; otherwise, it is important to soften the political barriers to the merger by revising the rule. In other words, in any case, it is necessary to focus on the relative sizes of the access cost and the degree of the heterogeneity when considering the revision of rules.

5. Concluding Remarks

This paper has constructed a model of the political merger of jurisdictions by incorporating the tradeoff between the scale economies and its costs such as the increases in the access cost to the facility, and in the heterogeneity among residents. Furthermore, by introducing the land market explicitly, our model includes the pecuniary spillover through the capitalization of difference in the policy among jurisdictions on the land rent. Due to this spillover and the difference in the preference intensity of residents, each jurisdiction has a different attitude toward the political merger. Specifically, as the residents have the stronger preference toward the public good, jurisdictions become more willing to merge with their neighbors because they receive more benefits from the merger through the scale economy and the internalization of the pecuniary spillover.

By using this model, we examine the efficiency of three alternative rules: such as i) Majority Voting; ii) Municipal Request; iii) Central Approval. We find that the Municipal Request becomes superior to the Majority Voting when the access cost is relatively low compared to the degree of the heterogeneity while the Majority Voting is superior to the Municipal Request when the access cost is relatively high. These results stem from the difference in the welfare gain of the economy through the merger and the gain or the loss of internalizing the pecuniary spillover for individual jurisdictions. In many real cases, the Municipal Request or the Central Approval is more applied to the merger of lower-tier jurisdictions compared to the Majority Voting in order to make the merger easier to accomplish. Our results suggest that since the access cost is decreasing due to the progress in the transportation technology, these two rules, the Municipal Request and the Central Approval, may become more efficient than ever. It is, however, shown that in some cases, these two rules may become inferior to the Majority Voting especially when the access cost is relatively high against the degree of the heterogeneity. That is, when choosing the rule for the political merger, it is important to care about the degree of the heterogeneity among households as well as the access cost.

Although it is difficult to observe the degree of the heterogeneity among households, the degree might be revealed through the difference in the public good provision among jurisdictions. Therefore, in the future research, we need to construct the empirical model to investigate the efficiency of alternative rules. In addition, since, in our model, the geographic configuration of towns is exogenously given, it is important to deal with the case where the geographic configuration of towns is also endogenously determined. Furthermore, in practice, the merger of the central city and its suburb is still a topic of debate; hence, we also need to work with the political merger in the monocentric city model as in Okamoto (2009).

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Appendix A: The Second Stage Outcomes

In order to derive the second stage outcomes, we treat the land rent at the periphery in one of jurisdictions as fixed. The second stage outcomes are summarized in the following tables:

<<TABLE A1: About HERE>>

By using the results summarized in Table A1, the net income under two configurations, σ_{12} and σ_{23} is computed as:

$$R_{12}(\sigma_{12}) = \frac{1}{3}\left[\left(\theta_{12}^m - \theta_3^m\right) + \theta_2^b \ln\frac{\theta_3^m}{2\theta_{12}^m}\right] + \frac{t\bar{N}}{6},$$

$$R_3(\sigma_{12}) = \frac{2}{3}\left[\left(\theta_3^m - \theta_{12}^m\right) + \theta_2^b \ln\frac{2\theta_{12}^m}{\theta_3^m}\right] - \frac{t\bar{N}}{6},$$

$$R_1(\sigma_{23}) = \frac{2}{3}\left[\left(\theta_1^m - \theta_3^m\right) + \theta_1^b \ln\frac{2\theta_{23}^m}{\theta_1^m}\right] - \frac{t\bar{N}}{6},$$

$$R_{23}(\sigma_{23}) = \frac{1}{3}\left[\left(\theta_{23}^m - \theta_1^m\right) + \theta_1^b \ln\frac{\theta_1^m}{2\theta_{23}^m}\right] + \frac{t\bar{N}}{6}.$$

Appendix B: The Proof of Proposition 1

Lemma 2

All three towns under the configuration σ_{ϕ} *prefer the configuration* σ_{23} *to* σ_{12}.

Proof:

There are two types of the two-town configuration, σ_{12} and σ_{23}, and we compare the payoffs of all towns between under σ_{12} and σ_{23}. Through the calculation,

$$V\left(\theta_1^m,\sigma_{12}\right)-V\left(\theta_1^m,\sigma_{23}\right)$$
$$=\frac{\theta}{18}\left[\left(\alpha-1\right)\ln\frac{\alpha+5}{4\left(\alpha+2\right)}+4\alpha\ln\frac{\left(5\alpha+1\right)}{4\left(2\alpha+1\right)}+2\ln\frac{\left(5\alpha+1\right)\left(\alpha+2\right)^3}{4\left(2\alpha+1\right)^4}\right]<0,$$

$$V\left(\theta_2^m,\sigma_{12}\right)-V\left(\theta_2^m,\sigma_{23}\right)$$
$$<\frac{\theta}{18}\left[9\left(\alpha+1\right)\ln\frac{\alpha+2}{2\alpha+1}+2\left(\alpha+2\right)\ln\frac{\left(\alpha+2\right)\left(\alpha+5\right)}{\left(2\alpha+1\right)\left(5\alpha+1\right)}\right]<0,$$

$$V\left(\theta_3^m,\sigma_{12}\right)-V\left(\theta_3^m,\sigma_{23}\right)$$
$$=\frac{\theta}{18}\left[\left(7\alpha-7\right)\ln\frac{5\alpha+1}{4\left(2\alpha+1\right)}+4\alpha\ln\frac{\left(\alpha+2\right)\left(5\alpha+1\right)^3}{16\left(2\alpha+1\right)^3\left(\alpha+5\right)}+4\ln\frac{\left(\alpha+2\right)^4\left(5\alpha+1\right)}{\left(2\alpha+1\right)^4\left(\alpha+5\right)}\right]<0.$$

Thus, all towns strictly prefer the configuration σ_{23} to σ_{12}: i.e. we can rule out the possibility that the configuration σ_{12} as the equilibrium configuration.

QED

Lemma 3

i) Towns 2 and 3 choose the isolation, $s_i^*=\{i\}$, *if* $t>t_i(\sigma_{23},\sigma_{\phi})$ *while it supports the* *merger of three towns,* $s_i^* = \{1,2,3\}$ *if* $t_i(\sigma_{123},\sigma_\phi) \geq t$. *For the domain of* $t_i(\sigma_{23},\sigma_\phi) \geq t > t_i(\sigma_{123},\sigma_\phi)$ *, they prefer the two-town merger.*

ii) Town 1 prefers the isolation, $s_1^* = \{1\}$ *, if* $t > t_i(\sigma_{123},\sigma_\phi)$ *; otherwise, they choose the three-town merger* $s_1^* = \{1,2,3\}$.

Proof:

By using Lemma 2, there are three types of the configuration, $\sigma_\phi, \sigma_{23},$ and σ_{123}, which might emerge at the equilibrium. By comparing the payoffs of three towns between two configurations, σ_ϕ and σ_{23}, we have thresholds $t_i(\sigma_{23},\sigma_\phi)$ (i=1, 2, 3):

$$t_1\left(\sigma_{23},\sigma_\phi\right) = \frac{6}{\bar{N}}\left[\frac{\theta_2^b}{3}\ln\frac{\theta_2^m}{\theta_3^m} + \frac{2\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_2^m}\right], \tag{B1-1}$$

$$t_2\left(\sigma_{23},\sigma_\phi\right) = \frac{6}{\bar{N}}\left[\theta_2^m\ln\frac{2\theta_{23}^m}{\theta_2^m} + \frac{\theta_2^b}{3}\ln\frac{\theta_2^m}{\theta_3^m} + \frac{\theta_1^b}{3}\ln\frac{\theta_2^m}{2\theta_{23}^m}\right], \tag{B1-2}$$

$$t_3\left(\sigma_{23},\sigma_\phi\right) = \frac{6}{\bar{N}}\left[\theta_3^m\ln\frac{2\theta_{23}^m}{\theta_3^m} + \frac{2\theta_2^b}{3}\ln\frac{\theta_3^m}{\theta_2^m} + \frac{\theta_1^b}{3}\ln\frac{\theta_2^m}{2\theta_{23}^m}\right]. \tag{B1-3}$$

Following the similar procedures, we obtain the thresholds $t_i(\sigma_{123},\sigma_\phi)$ and $t_i(\sigma_{123},\sigma_{23})$:

$$t_1\left(\sigma_{123},\sigma_\phi\right) = \frac{2}{\bar{N}}\left[\theta_1^m\ln\frac{3\theta_{123}^m}{\theta_1^m} + \frac{2\theta_1^b}{3}\ln\frac{\theta_1^m}{\theta_2^m} + \frac{\theta_2^b}{3}\ln\frac{\theta_2^m}{\theta_3^m}\right], \tag{B2-1}$$

$$t_2\left(\sigma_{123},\sigma_\phi\right) = \frac{2}{\bar{N}}\left[\theta_2^m\ln\frac{3\theta_{123}^m}{\theta_2^m} + \frac{\theta_1^b}{3}\ln\frac{\theta_2^m}{\theta_1^m} + \frac{\theta_2^b}{3}\ln\frac{\theta_2^m}{\theta_3^m}\right], \tag{B2-2}$$

$$t_3\left(\sigma_{123},\sigma_{\phi}\right)=\frac{2}{\bar{\bar{N}}}\left[\theta_3^m \ln\frac{3\theta_{123}^m}{\theta_3^m}+\frac{\theta_1^b}{3}\ln\frac{\theta_2^m}{\theta_1^m}+\frac{2\theta_2^b}{3}\ln\frac{\theta_3^m}{\theta_2^m}\right], \tag{B2-3}$$

$$t_1\left(\sigma_{123},\sigma_{23}\right)=\frac{3}{\bar{\bar{N}}}\left[\theta_1^m \ln\frac{3\theta_{123}^m}{\theta_1^m}+\frac{2\theta_1^b}{3}\ln\frac{\theta_1^m}{2\theta_{23}^m}\right], \tag{B3-1}$$

$$t_i\left(\sigma_{123},\sigma_{23}\right)=\frac{3}{\bar{\bar{N}}}\left[\theta_i^m \ln\frac{3\theta_{123}^m}{2\theta_{23}^m}+\frac{2\theta_1^b}{3}\ln\frac{2\theta_{23}^m}{\theta_1^m}\right], \text{ for } i\text{=2, 3.} \tag{B3-2}$$

For each town, by comparing Equations (B1), (B2), and (B3), we have:

$$t_i\left(\sigma_{123},\sigma_{\phi}\right)=\frac{2t_i\left(\sigma_{123},\sigma_{23}\right)+t_i\left(\sigma_{23},\sigma_{\phi}\right)}{3} \text{ for } i\text{=1, 2, 3.} \tag{B4}$$

Therefore, we can limit our focus on the comparison of two of three thresholds:

$$t_1\left(\sigma_{23},\sigma_{\phi}\right)-t_1\left(\sigma_{123},\sigma_{\phi}\right)$$
$$=\frac{\theta}{9\bar{\bar{N}}}\left[\alpha\ln\frac{(8\alpha+4)^{12}}{(5\alpha+1)^{8}(\alpha+5)^{3}(9\alpha+9)}+\ln\frac{(8\alpha+4)^{24}(\alpha+5)^{7}}{(5\alpha+1)^{4}(3\alpha+3)^{12}(9\alpha+9)^{15}}\right]<0,$$

$$t_2\left(\sigma_{23},\sigma_{\phi}\right)-t_2\left(\sigma_{123},\sigma_{\phi}\right)$$
$$=\frac{\theta}{9\bar{\bar{N}}}\left[\alpha\ln\frac{(8\alpha+4)^{21}(\alpha+5)^{2}}{27(3\alpha+3)^{15}(5\alpha+1)^{8}}+\ln\frac{(8\alpha+4)^{15}(\alpha+5)^{4}}{27(3\alpha+3)^{15}(5\alpha+1)^{4}}\right]>0,$$

$$t_3\left(\sigma_{23},\sigma_{\phi}\right)-t_3\left(\sigma_{123},\sigma_{\phi}\right)=$$
$$\frac{\theta}{9\bar{\bar{N}}}\left[\alpha\ln\frac{(8\alpha+4)^{39}(\alpha+5)^{2}}{3^{15}(5\alpha+1)^{14}(3\alpha+3)^{27}}+\ln\frac{(5\alpha+1)^{2}(\alpha+5)^{4}}{27(3\alpha+3)^{3}(8\alpha+4)^{3}}\right]>0.$$

In summary, for each town, we have the following relationship:

$$t_1\left(\sigma_{123},\sigma_{23}\right)>t_1\left(\sigma_{123},\sigma_{\phi}\right)>t_1\left(\sigma_{23},\sigma_{\phi}\right), \tag{B5-1}$$

$$t_i\left(\sigma_{23},\sigma_{23}\right)>t_i\left(\sigma_{123},\sigma_{\phi}\right)>t_i\left(\sigma_{123},\sigma_{\phi}\right) \text{ for } i\text{=2, 3.} \tag{B5-2}$$

According to the definition of $t_i(\sigma_y,\sigma_{y'})$, Towns 2 and 3 change the strategy for the merger: namely, as *t* increases, the three-town merger to the isolation via the two-town merger. In contrast, under the configurations, σ_{23} and σ_{ϕ}, Town 1 does not merge with its neighbor; therefore, Town 1 chooses the isolation as long as $t > t_1(\sigma_{123}, \sigma_{23})$.

QED

Lemma 4

Among three towns, Town 1's threshold, $t_1(\sigma_y, \sigma_{y'})$*, is the lowest while Town 3's,* $t_3(\sigma_y, \sigma_{y'})$*, is the highest for any pair of configurations,* σ_y *and* $\sigma_{y'}$*.*

Proof:

According to the comparison of Equations (B1) and (B2), we have:

$$t_1\left(\sigma_{23}, \sigma_{\phi}\right) < t_2\left(\sigma_{23}, \sigma_{\phi}\right) < t_3\left(\sigma_{23}, \sigma_{\phi}\right), \tag{B6-1}$$

$$t_1\left(\sigma_{123}, \sigma_{23}\right) < t_2\left(\sigma_{123}, \sigma_{23}\right) < t_3\left(\sigma_{123}, \sigma_{23}\right). \tag{B6-2}$$

According to Equations (B4) and (B6), it is easily shown that:

$$t_1\left(\sigma_{123}, \sigma_{\phi}\right) < t_2\left(\sigma_{123}, \sigma_{\phi}\right) < t_3\left(\sigma_{123}, \sigma_{\phi}\right).$$

QED

Proposition 1

The configuration σ_{ϕ} *is the equilibrium of the Majority Voting if* $t > t^E(\sigma_{23}, \sigma_{\phi})$*; the configuration* σ_{23}*, if* $t^E(\sigma_{23}, \sigma_{\phi}) \geq t > t^E(\sigma_{123}, \sigma_{23})$*;* σ_{23}*, if* $t^E(\sigma_{123}, \sigma_{23}) \geq t$*. Formally,*

$$\sigma^E = \begin{cases} \sigma_\phi \text{ for } t > t^E\left(\sigma_{23}, \sigma_\phi\right), \\ \sigma_{23} \text{ for } t^E\left(\sigma_{23}, \sigma_\phi\right) \geq t > t^E\left(\sigma_{123}, \sigma_{23}\right), \\ \sigma_{123} \text{ for } t^E\left(\sigma_{123}, \sigma_{23}\right) \geq t. \end{cases}$$

Proof:

By using the definition of $t^E(\sigma_y, \sigma_{y'})$ and Equations (B6), it is easily derived:

$$t^E\left(\sigma_{23}, \sigma_\phi\right) = t_2\left(\sigma_{23}, \sigma_\phi\right),$$

$$t^E\left(\sigma_{123}, \sigma_{23}\right) = t_1\left(\sigma_{123}, \sigma_{23}\right).$$

According to Lemma 3, we have:

$$s_i^* = \{2,3\} \text{ for } i = 2,3 \text{ if } t^E\left(\sigma_{23}, \sigma_\phi\right) \geq t,$$

$$s_i^* = \{1,2,3\} \text{ for } i = 1,2,3 \text{ if } t^E\left(\sigma_{123}, \sigma_{23}\right) \geq t.$$

Finally, we need to consider when the configuration changes from σ_{23} to σ_{123}. From Lemma 4,

$$t^E\left(\sigma_{123}, \sigma_{23}\right) = t_1\left(\sigma_{123}, \sigma_{23}\right) < t_2\left(\sigma_{123}, \sigma_{23}\right) < t_3\left(\sigma_{123}, \sigma_{23}\right).$$

This implies that even if $t > t^E(\sigma_{123}, \sigma_{23})$, there may exist some of towns prefer the three-town merger to the two-town. Since the merger takes place only when all towns participating approve it, for $t > t^E(\sigma_{123}, \sigma_{23})$, Town 1 never supports the three-town merger; therefore, the two-town merger is realized if $t > t^E(\sigma_{123}, \sigma_{23})$.

QED

Appendix C: The Proofs of Propositions 4 and 5

Proposition 4

The configuration σ_ϕ *maximizes the social surplus if* $t > t^O(\sigma_{23}, \sigma_\phi)$ *; the configuration* σ_{23} *, if* $t^O(\sigma_{23}, \sigma_\phi) \geq t > t^O(\sigma_{123}, \sigma_{23})$ *;* σ_{23} *, if* $\alpha = \bar{\theta} / \underline{\theta}$ *.*

Formally,

$$\sigma^O = \begin{cases} \sigma_\phi \text{ for } t > t^O\left(\sigma_{23}, \sigma_\phi\right), \\ \sigma_{23} \text{ for } t^O\left(\sigma_{23}, \sigma_\phi\right) \geq t > t^O\left(\sigma_{123}, \sigma_{23}\right), \\ \sigma_{123} \text{ for } t^O\left(\sigma_{123}, \sigma_{23}\right) \geq t. \end{cases}$$

Proof:

First, we can rule out the possibility of emerging the configuration σ_{12} as the welfare maximizing configuration since we have the following relation:

$$SW\left(\sigma_{12}\right) = \bar{N}\sum_{i=1}^{3} V\left(\theta_i^m, \sigma_{12}\right) < \bar{N}\sum_{i=1}^{3} V\left(\theta_i^m, \sigma_{23}\right) = SW\left(\sigma_{23}\right).$$

Thus, we have three candidates, σ_ϕ, σ_{23}, and σ_{123}, for the welfare maximizing configuration. By comparison of social welfares and substituting $\bar{\theta} = \alpha\underline{\theta}$, we obtain:

$$t^O\left(\sigma_{123}, \sigma_\phi\right) = \frac{2}{3\bar{N}}\left[\theta_1^m \ln\frac{9(\alpha+1)}{\alpha+5} + \theta_2^m \ln 3 + \theta_3^m \ln\frac{9(\alpha+1)}{5\alpha+1}\right],$$

$$t^O\left(\sigma_{123}, \sigma_{23}\right) = \frac{1}{\bar{N}}\left[\theta_1^m \ln\frac{9(\alpha+1)}{\alpha+5} + \sum_{j=2}^{3}\theta_j^m \ln\frac{9(\alpha+1)}{4(2\alpha+1)}\right],$$

$$t^O\left(\sigma_{23}, \sigma_\phi\right) = \frac{2}{\bar{N}}\left[\theta_2^m \ln\frac{4(2\alpha+1)}{3(\alpha+1)} + \theta_3^m \ln\frac{4(2\alpha+1)}{5\alpha+1}\right].$$

According to the comparison of those thresholds,

$$t^O\left(\sigma_{123},\sigma_\phi\right)=\frac{2t^O\left(\sigma_{123},\sigma_{23}\right)+t^O\left(\sigma_{23},\sigma_\phi\right)}{3}. \tag{C1}$$

Therefore, we can limit our focus on thresholds $t^O\left(\sigma_{23},\sigma_\phi\right)$ and $t^O\left(\sigma_{123},\sigma_{23}\right)$. From the comparison of these two thresholds:

$$t^O\left(\sigma_{23},\sigma_\phi\right)-t^O\left(\sigma_{123},\sigma_{23}\right)=\frac{1}{\bar{N}}\left[-\theta_1^m\ln\frac{3^8\left(\alpha+1\right)^5}{4^3\left(\alpha+5\right)^2\left(2\alpha+1\right)^3}+\theta_3^m\ln\frac{4^6\left(2\alpha+1\right)^6}{3^6\left(5\alpha+1\right)^2\left(\alpha+1\right)^4}\right].$$

(C2)

The terms within the logarithm function of (B2) satisfy:

$$1<\frac{3^8\left(\alpha+1\right)^5}{4^3\left(\alpha+5\right)^2\left(2\alpha+1\right)^3}<\frac{4^6\left(2\alpha+1\right)^6}{3^6\left(5\alpha+1\right)^2\left(\alpha+1\right)^4}.$$

Therefore,

$$t^O\left(\sigma_{23},\sigma_\phi\right)-t^O\left(\sigma_{123},\sigma_{23}\right)>\frac{\left(\alpha-1\right)\underline{\theta}}{3\bar{N}}\ln\frac{3^8\left(\alpha+1\right)^5}{4^3\left(\alpha+5\right)^2\left(2\alpha+1\right)^3}>0. \tag{C3}$$

Hence, according to Equations (B1) and (B3), we obtain Proposition 1.

QED

Proposition 5

The threshold $t^T(\sigma_y,\sigma_{y'})$ *has the following relationship:*

$$t^E\left(\sigma_{123},\sigma_{23}\right)<t^O\left(\sigma_{123},\sigma_{23}\right)<t^C\left(\sigma_{123},\sigma_{23}\right)<t^R\left(\sigma_{123},\sigma_{23}\right), \tag{22-1}$$

$$t^E\left(\sigma_{23},\sigma_\phi\right)=t^C\left(\sigma_{23},\sigma_\phi\right)<t^O\left(\sigma_{23},\sigma_\phi\right)<t^R\left(\sigma_{23},\sigma_\phi\right). \tag{22-2}$$

Proof:

From the calculation, $t^O(\sigma_y, \sigma_{y'})$ and $t_i(\sigma_y, \sigma_{y'})$ have the following relation:

$$t^O\left(\sigma_y, \sigma_{y'}\right) = \frac{1}{3}\sum_{i=1}^{3} t_i\left(\sigma_y, \sigma_{y'}\right). \tag{C4}$$

Let us first focus on Eq. (22-1). According to Lemma 4, we have:

o

$$t^E\left(\sigma_{123}, \sigma_{23}\right) = t_1\left(\sigma_{123}, \sigma_{23}\right) < t^C\left(\sigma_{123}, \sigma_{23}\right) = t_2\left(\sigma_{123}, \sigma_{23}\right) < t^R\left(\sigma_{123}, \sigma_{23}\right) = t_3\left(\sigma_{123}, \sigma_{23}\right).$$

By using (C4),

$$t^E\left(\sigma_{123}, \sigma_{\phi}\right) < t^O\left(\sigma_{123}, \sigma_{\phi}\right) < t^R\left(\sigma_{123}, \sigma_{\phi}\right).$$

Therefore, we focus on the comparison of $t^O(\sigma_{123}, \sigma_{\phi})$ and $t^C(\sigma_{123}, \sigma_{\phi})$:

$$t^O\left(\sigma_{123}, \sigma_{23}\right) - t^E\left(\sigma_{123}, \sigma_{23}\right) = \frac{(\alpha-1)\underline{\theta}}{3\bar{N}}\left[3\ln\frac{9(\alpha+1)}{4(2\alpha+1)} + \ln\frac{4(2\alpha+1)}{\alpha+5}\right].$$

In case of Eq. (22-2), by using Lemma 4 and (C4), we have:

$$t^E\left(\sigma_{23}, \sigma_{\phi}\right) = t^C\left(\sigma_{23}, \sigma_{\phi}\right) = t_2\left(\sigma_{23}, \sigma_{\phi}\right) < t^A\left(\sigma_{23}, \sigma_{\phi}\right) = t_3\left(\sigma_{23}, \sigma_{\phi}\right),$$

$$t^O\left(\sigma_{23}, \sigma_{\phi}\right) < t^A\left(\sigma_{23}, \sigma_{\phi}\right).$$

Furthermore, through the calculation, we obtain:

$$t^O\left(\sigma_{23}, \sigma_{\phi}\right) - t^E\left(\sigma_{23}, \sigma_{\phi}\right) = \frac{\alpha-1}{3\bar{N}}\ln\frac{4(2\alpha+1)}{5\alpha+1}.$$

QED

FIGURES

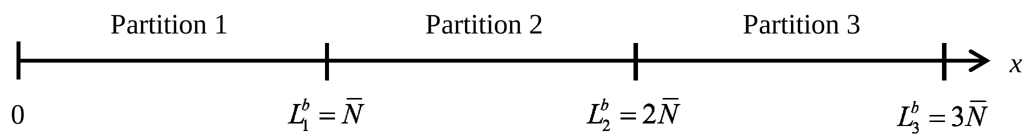

Figure 1: Geographic Configuration.

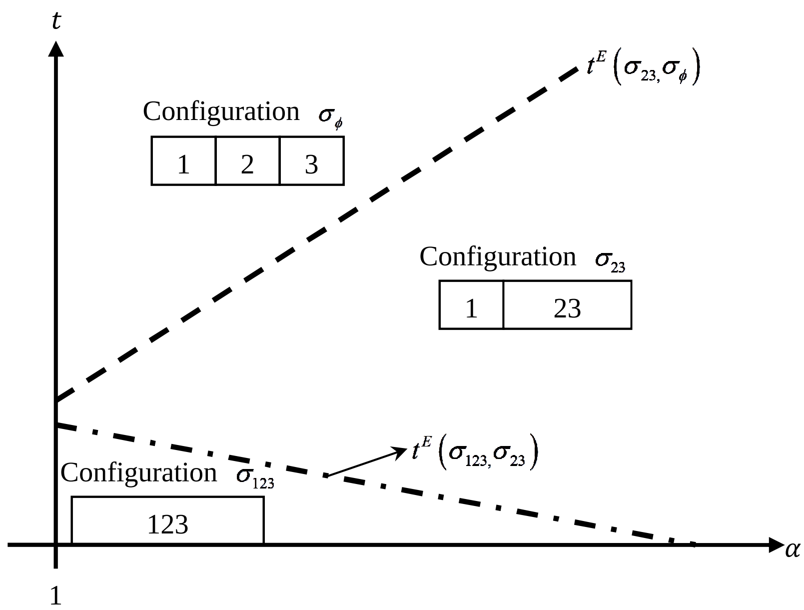

Figure 2: the Equilibrium Configuration under the Geographic Pattern {1, 2, 3}.

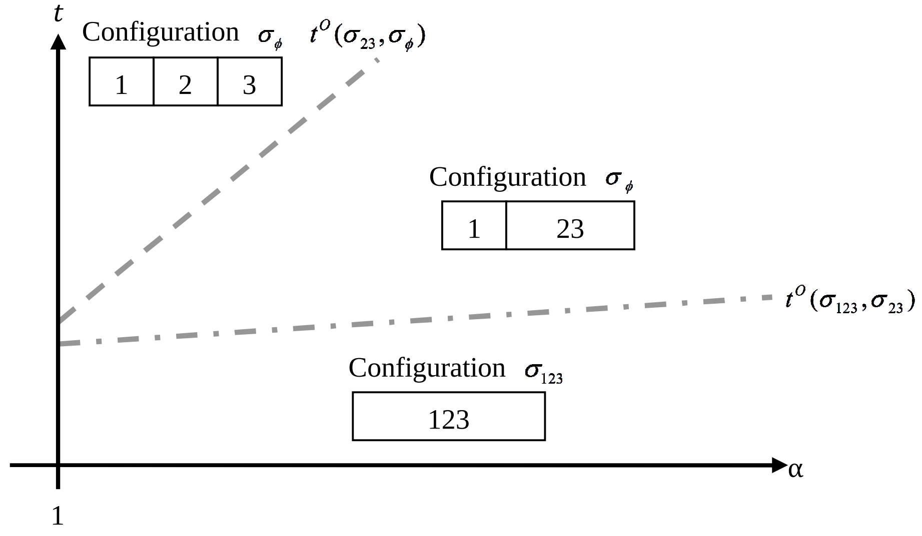

Figure 3: the Welfare-Maximizing Configuration

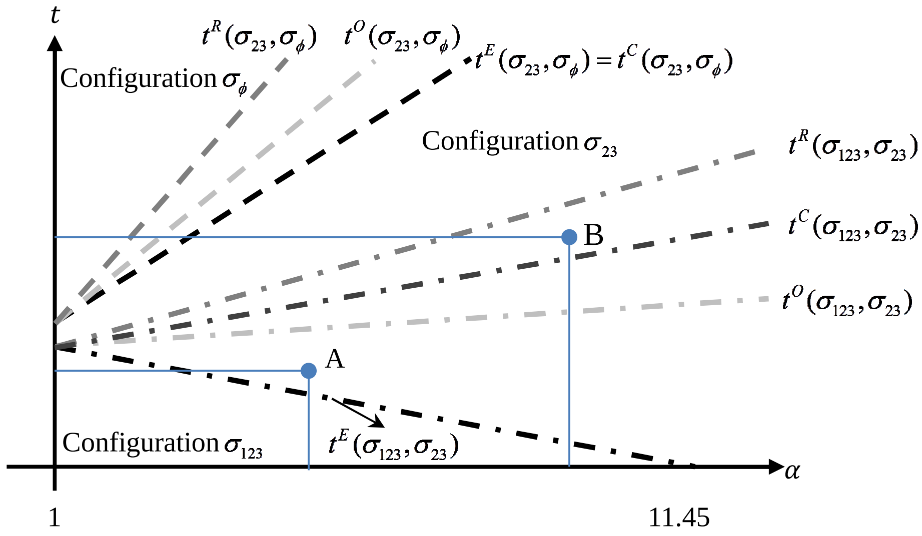

Figure 4: Comparison of Alternative Rules

TABLES

Table 1: Rules Implemented in the United States

Rules	Requirement for Reorganization	Number of States*
Judicial Approval	Approval of reorganization proposals by courts	1
Legislative Approval	Approval of reorganization proposals by legislatures of upper-tier governments	8
Quasi-Legislative Approval	Establishment of state-authorized independent commissions and approval by commissions	8
Majority Voting	Referendums: voting groups include residents in jurisdictions to be reorganized	29
Municipal Request	Setup of proposal by a jurisdiction and approval by the legislatures of this jurisdiction	7

Source: Lindsey (2004).

*: Total number of states exceeds 50 since three states report two rules are implemented.

Table 2: Possible Merger Patterns

Configuration	Towns		
σ_{ϕ}	Town 1	Town 2	Town 3
σ_{12}	Town 12		Town 3
σ_{23}	Town 1	Town 23	
σ_{123}	Town 123		

Table A1: the Head Tax, Public Good, and Land Rent at the Periphery

Configuration	Towns	Tax	Public Good	Land Rent at the Periphery
$\boldsymbol{\sigma_{12}}$	Town 12	θ_{12}^m	$\dfrac{2\theta_{12}^m \bar{N}}{c}$	$\theta_3^m - \theta_{12}^m + \theta_2^b \ln \dfrac{2\theta_{12}^m}{\theta_3^m} + \bar{r}_3 - \dfrac{t\bar{N}}{6}$
	Town 3	θ_3^m	$\dfrac{\theta_3^m \bar{N}}{c}$	$\bar{r}_3$
$\boldsymbol{\sigma_{23}}$	Town 1	θ_1^m	$\dfrac{\theta_1^m \bar{N}}{c}$	$\bar{r}_1$
	Town 23	θ_{23}^m	$\dfrac{2\theta_{23}^m \bar{N}}{c}$	$\theta_1^m - \theta_{23}^m + \theta_1^b \ln \dfrac{2\theta_{23}^m}{\theta_1^m} + \bar{r}_1 - \dfrac{t\bar{N}}{6}$
$\boldsymbol{\sigma_{123}}$	Town 123	θ_{123}^m	$\dfrac{3\theta_{123}^m \bar{N}}{c}$	$\bar{r}_{123}$